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Regularization of soft-X-ray imaging in the DIII-D tokamak

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ABSTRACT

An image inversion scheme for the soft-X-ray imaging system (SXRIS) diagnostic at the DIII-D tokamak is developed to obtain the local soft-X-ray emission at a poloidal cross-section from the spatially line-integrated image taken by the SXRIS camera. The scheme uses the Tikhonov regularization method since the inversion problem is generally ill-posed. The regularization technique uses the generalized singular value decomposition (GSVD) to determine a solution that depends on a free regularization parameter. The latter has to be chosen carefully, and the so-called *L-curve* method to find the optimum regularization parameter is outlined. A representative test image is used to study the properties of the inversion scheme with respect to inversion accuracy, amount/strength of regularization, image noise and image resolution. The optimum inversion parameters are identified, while the L-curve method successfully computes the optimum regularization parameter. Noise is found to be the most limiting issue, but sufficient regularization is still possible at noise to signal ratios up to 10%–15%. Finally, the inversion scheme is applied to measured SXRIS data and the line-integrated SXRIS image is successfully inverted.

1. Introduction

Methods for computed tomography are used in many fields like medicine, nondestructive materials testing, geophysics, atmospheric research and have also been applied to magnetically confined fusion plasmas [1,2]. The common characteristic is that many nonlocal measurements, such as line integrals, are processed mathematically to estimate local physical quantities, such as emissivity or absorption.

Fusion has a long history of soft-X-ray (SXR) detection in the plasma core of tokamaks [3] and stellarators to e.g. detect core MHD modes [4], diagnose core islands [5] or analyze impurity concentrations [6]. A number of tomography methods, which have found applicability to SXR, bolometer, and other tomography diagnostics in magnetically confined fusion plasmas, have been used to various levels of success. Those methods have been limited in spatial resolution and/or cross-detector calibrations [7]. Tangential imaging has historically given better spatial resolution, but interpretation of the images is nontrivial due to the 3-D line-of-sight integration and requires advanced inversion techniques [8,9], like the Tikhonov regularization [10].

A tokamak confines the plasma inside a torus-shaped vacuum chamber by a strong magnetic field. The magnetic field of a tokamak equilibrium is given by a strong toroidal magnetic field and the poloidal field, generated by the strong toroidal plasma current. The resulting total field is helical in nature; we will exploit this helical property later in the paper. The

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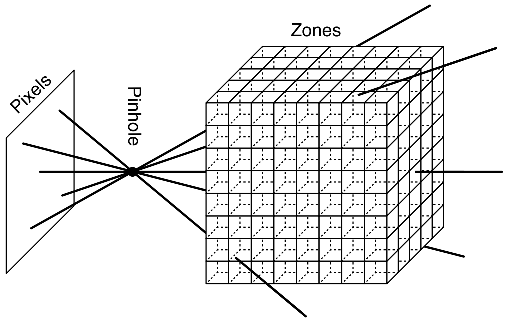

Fig. 1. Schematic camera setup for recording line-integrated SXR emission. Several line-of-sights, starting at some camera pixels and going through the pinhole, penetrate the emission volume, which is divided into multiple zones.

field lines are organized on surfaces of constant poloidal magnetic flux ψ, the so-called *flux surfaces*. All field lines on such a flux surface wind themselves helically around the torus and have a constant ratio of toroidal to poloidal rotations, the so-called *safety factor q*. Typically q is monotonically increasing from the center towards the separatrix, which is the last closed surface that then determines the plasma boundary and separates the closed field lines inside from the open ones in the so-called *scrape-off layer*. The flux surface configuration is then toroidally axisymmetric. This symmetry allows to project volume quantities, like SXR emission, into a single plane, the tangency plane; we will make use of this later in the paper. Here we focus on the DIII-D tokamak [11].

The soft-X-ray imaging system (SXRIS) [12] at DIII-D is a tangential camera that measures the line-integrated SXR emission in the plasma edge region. The system uses pinhole optics focusing radiation onto a scintillator, which is imaged with visible optics. It is used to diagnose structural changes due to plasma response [13] to externally applied resonant magnetic perturbations (RMPs). The diagnostic can use different energy filters to discriminate between different spatial regions. Within the steep gradient region of the H-mode pedestal [14], helical displacements of up to 5 cm, which change with the poloidal angle are found, which can be modeled and explained through a synthetic diagnostic [15]. To interpret the measured data, the Tikhonov regularization, found to be applicable to the SXRIS [16], is used to obtain stabilized solutions to the ill-posed problem of inverting line-integrated images.

So far, SXR imaging was used only in the plasma core to detect large scale structures. Here we apply it to the perturbed tokamak plasma edge region for the first time. In this work we determine the requirements for the Tikhonov regularization and develop an algorithm to find high resolution and high quality image inversions. The latter are needed to identify and characterize features with potentially low signal to noise and high spatial structure, which are predicted by plasma theory to appear in the perturbed plasma edge.

In the next section we give an overview on the Tikhonov regularization method and construct its solution using the generalized singular value decomposition (GSVD). The method using the GSVD is equivalent to the method developed in Ref. [10] and applied in Refs. [5,4], but does not require the direct inversion of the side constraint operator. The setup of the SXR image inversion problem is discussed in Section 3. The side constraints as well as the optimum regularization parameter is discussed. In Section 4 we apply the regularization to an analytic test image and study the inversion quality with respect to noise and resolution. An application to real SXRIS data is demonstrated in Section 5. The conclusions summarize the quality of the inversion method and give suggestions on future improvements in the final section.

2. Mathematical approach to high resolution image inversion

Mathematically, the 3-D volume of SXR emission is divided into $n = n_x n_y n_z$ emission zones. Each zone is a cube of size dr^3 inside the volume. On one side of the volume is a pinhole camera, which records the SXR emission. For simplicity we neglect finite pinhole size effects and assume that the pinhole diameter is infinitesimally small. Note that later on we will reduce the emission volume to 2-D by exploiting a system symmetry. The camera itself is represented by a 2-D "image" which has m number of pixels. Each pixel of the image has exactly one line-of-sight through the pinhole and records all emission from the volume along its line-of-sight. Fig. 1 shows a schematic sketch of the described setup.

The relation between local emission, the emission in each zone of the volume, $\boldsymbol{\varepsilon} \in \mathbb{R}^n$ and line-integrated emission, the value at each pixel (literally a camera pixel) in the image plane, $\mathbf{s} \in \mathbb{R}^m$ is given by the linear equation

$$\mathbf{L} \cdot \boldsymbol{\varepsilon} = \mathbf{s} \tag{1}$$

with the geometric transform $m \times n$ matrix $\mathbf{L}$, which will be discussed in more detail in Section 3.

2.1. Tikhonov regularization approach

In Eq. (1) we use $m > n$ so the system is overdetermined. The solution to this system of equations is given by