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Time-dependent current source identification for numerical simulations of Maxwell's equations

J. Benoit [a,b,*], C. Chauvière [c], P. Bonnet [b]

[a] CEA, LIST, F-91191 Gif-sur-Yvette, France
[b] Clermont University, Blaise Pascal University, CNRS, UMR 6602, Pascal Institute, 63171 Aubière Cedex, France
[c] Clermont University, Blaise Pascal University, CNRS, UMR 6620, LM, 63171 Aubière Cedex, France

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ABSTRACT

This paper discusses an inverse source problem for time-dependent linear Maxwell's equations. We propose a numerical procedure to compute a current density source that gives a specified electromagnetic field. The efficiency of our method is shown by numerical tests, first in 1D and then we move on to a 3D example with antennas. In that case, we show potential applications for fault detection.

1. Introduction

Inverse problems for Maxwell's equations may be used to connect physical parameters describing a model to collected observations, e.g. near or far electromagnetic fields, currents, voltages. The purpose of such studies may be the non-destructive characterization of media, the optimization of material properties, the identification of current source distribution, the determination of conductivity or the shape of an obstacle. Typical applications are in the fields of medical imaging [1], power engineering [2], radar and antennas, remote sensing [3] or electromagnetic compatibility [5–7]. If the solution of inverse problems is conceptually harder to solve than the forward model, the number and the importance of scientific and industrial applications justify efforts in developing mathematical and numerical methods. A huge literature deals with inverse scattering problems in electromagnetism both in time-dependent and harmonic domains. For instance, to reconstruct the shape (or a local inhomogeneity) from multistatic data at a fixed frequency, the linear sampling method (LSM) may be an effective method [8]. Yet, the reconstruction is sensitive to the choice of the polarization vector, the frequency, the set of observation points and in practice a regularization process is used to compute a nearby solution.

Most of the time, the frequency domain inverse scattering approach is adopted for its direct connection with frequential experimental measurements, and more straightforward theoretical development. In time-harmonic simulations, any point of the medium impacts the whole electromagnetic field. In contrast, for short time simulations, time-domain approaches can exploit causality to limit the computational domain, potentially reducing the number of unknowns. Moreover, data in time domain are richer in information because of the large frequency spectrum of temporal signals.

* Corresponding author at: Clermont University, Blaise Pascal University, CNRS, UMR 6602, Pascal Institute, 63171 Aubière Cedex, France.
E-mail addresses: jaume.benoit@cea.fr (J. Benoit), cedric.chauviere@math.univ-bpclermont.fr (C. Chauvière), pierre.bonnet@lasmea.univ-bpclermont.fr (P. Bonnet).

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Using optimization-based algorithms and a cost function to be minimized, studies have shown the efficiency of such approach [9,10], mainly for two dimensional problems. In this paper we are concerned with a time domain inverse scattering problem. Time dependent Maxwell's equations appear in different forms, such as

$$\varepsilon\frac{\partial\mathbf{E}}{\partial t} = \nabla\times\mathbf{H} - \sigma\mathbf{E} - \mathbf{J} \tag{1}$$

$$\mu\frac{\partial\mathbf{H}}{\partial t} = -\nabla\times\mathbf{E} \tag{2}$$

Where $\mathbf{E}$ $\left(\mathrm{V\,m^{-1}}\right)$ is the electric field and $\mathbf{H}$ $\left(\mathrm{A\,m^{-1}}\right)$ the magnetic field. In the forward problem, the electric current density $\mathbf{J}$ $\left(\mathrm{A\,m^{-1}}\right)$ is a given source term and ε, μ and σ are (tensors) coefficients representing the dielectric permittivity, the magnetic permeability and the conductivity, respectively. Here we have neglected divergence constraints since when they are satisfied initially, they will remain fulfilled forever. The equations in this paper are one or three-dimensional and they are provided with appropriate boundary conditions. Then, Eqs. (1)–(2) can be discretized (see [11–13], for example) to obtain a numerical approximation of the electromagnetic field in the computational domain.

In some inverse scattering problems, one aims at recovering the shape or the physical properties of an object of interest from measurements of the scattered fields solutions of Eqs. (1)–(2) [3,4]. If the unknown parameter is the electric current density $\mathbf{J}$, we address the so-called inverse source problem for Maxwell's equations [14,2]. A wide range of industrial problems take advantage of such studies. In practice, it might be a fruitful tool in the general frame of computing an equivalent model from a complex system. Based on the equivalence principle [15], a lot has been done in the frequency domain to transform near field to equivalent currents for radiating bodies. For example, non-iterative methods, based on boundary integrals using weighting functions, are developed in [16] for time-harmonic Maxwell's equations and Helmholtz's equation for the identification of dipolar sources. More recently, genetic algorithm-based methods reported in [17–19] have been applied to find an equivalent set of elemental electric and magnetic dipoles. For low frequency approximation (static or quasi-static case), inverse source problem for eddy current equations are used for non-destructive characterization and biomedical engineering [20].

Here, we are concerned with time dependent Maxwell's equations and, for the best of our knowledge, nothing has been done for addressing an inverse source problem in such a case. The problem we solve can be formulated as follows: assume that at a given point $\mathbf{x}_a$ of the domain, we want to obtain a specified electromagnetic field $(\mathbf{E}(\mathbf{x}_a,t),\mathbf{H}(\mathbf{x}_a,t))_{t\in I\subset\mathbb{R}^+}$. Our goal is to compute the discrete source $\mathbf{J}$ that produces such an electromagnetic field. The discrete source $\mathbf{J}$ can be expressed as an n element vector such that we are concerned with a discrete inverse source problem. In the particular case where the target field is specified only at one instant, an easy and elegant way to solve this inverse source problem is to use time reversal techniques [21]. Time reversal advantages, limitations and applications have been extensively studied by different research groups [22,23]. More specifically, time reversal works well for lossless media and is good at refocusing waves obtained by sources or scatterers having small supports in the physical domain. However, good refocusing might require multiple measurement times [23] and may also require many measurement points. In [24], a competitive and more general method called Linear Combination of Configuration Fields (LCCF), that avoids many of the limitations described above, was introduced. In that paper, numerical examples showed the superiority of the LCCF, especially when the target field has a complex shape with large support over the domain.

Based on the LCCF principle, we propose in this paper an extension of this idea to an electromagnetic target field of arbitrary duration. As for most inverse problems, the one considered here is ill-posed, i.e., small errors in the data field can produce large errors in the current source. Several types of regularization can be used to overcome the ill-posedness. Because physically acceptable solutions must have limited energy we choose in this paper the Tychonoff regularization [25]. Another drawback of inverse problem is non-uniqueness. Theoretical considerations such as the ill-posedness or uniqueness will not be considered in this paper but readers interested in these aspects can refer to [26,27]. In our case, we are interested in finding at least one current density source $\mathbf{J}$. Constraints could be added, to drive the solution to the inverse problem to satisfy some properties, but this is not the topic of this paper.

The paper is organized as follows: first, the physical governing equation are given and the problem to be solved is put in equations. The principle of the Linear Combination of Configuration Field is adapted for solving this inverse problem. Section 3 is devoted to numerical examples. The proposed method is first tested on an academic 1D example. Then we move on to a complex 3D case. This example shows that the method allows to annihilate the electromagnetic field generated by a wireless network settled in the living room, in the bed room, thanks to a second field based, for example, in the kitchen. The behavior of the method is studied with respect to small variations in the physical properties of the computational domain. We also show how the idea of the LCCF can be adapted experimentally. Finally, in Section 4 we draw some conclusions.

2. Description of the LCCF method

Assume that an electromagnetic field $(\mathbf{E}_1,\mathbf{H}_1) = (\mathbf{E}_1(\mathbf{x},t),\mathbf{H}_1(\mathbf{x},t))$, $(\mathbf{x},t)\in\Omega\times[0,t_{p+f}]$ propagates into a domain Ω and that its evolution in time and space can be described by Maxwell's equations

$$\varepsilon\frac{\partial\mathbf{E}_1}{\partial t} = \nabla\times\mathbf{H}_1 - \sigma\mathbf{E}_1 - \mathbf{J}_1 \tag{3}$$