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Short note

A simple and conservative operator-splitting semi-Lagrangian volume-of-fluid advection scheme

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ABSTRACT

A semi-Lagrangian, operator-splitting, high-performance volume-of-fluid advection scheme (SLOSH-VOF) is developed. In the backward time integration a directional splitting strategy is adopted, which greatly simplifies the definition and locating of the departure volume and reduces it to a grid cell expanded or compressed in one grid direction. The VOF value in the departure cell is estimated using a geometrical interpolation algorithm with a piecewise linear interface calculation (PLIC) scheme for the interfacial cells. The proposed SLOSH-VOF method is unconditionally stable and very large time steps can be used to significantly speed up the overall computations. It is conceptually simple and can be very easily implemented due to the direction-split advection. Its performance is evaluated through several benchmark advection tests. The SLOSH-VOF results are comparable to those from an Eulerian VOF method in terms of interface position errors, and improved with regard to mass conservation, even with CFL numbers increased up to one order of magnitude.

1. Introduction

A large amount and range of spatial resolution is needed for the numerical simulations of free-surface flows with wave breaking. Spatial resolution requirements can be alleviated by domain decomposition in parallel computation. Another challenge of such simulations is that the velocity can be huge in some regions where wave breaking occurs and spray forms. In explicit Eulerian schemes, the maximum time step is subject to the Courant–Friedrichs–Lewy (CFL) condition. Although in most regions the velocities are not high and the local CFL numbers are small, the overall time step can still be extremely small due to the regions with strong interface topological changes. This leads to a prohibitive computational cost. The computational efficiency also decreases when local grid refinement is used even though the velocity in the whole computational domain is uniform, since the maximum time step is determined by the region with the finest grid. Fully implicit schemes can relax the time step constraints, but they usually need Newton iterations and non-symmetric solvers. The Lagrangian method is unconditionally stable, but frequent remeshing is needed since the regular mesh distorts rapidly, which increases the computational efforts and leads to the loss of accuracy.

The semi-Lagrangian method, which takes advantage of both the Lagrangian and Eulerian approaches, has been popularly used for numerical weather predictions in the meteorological community [12]. In the semi-Lagrangian method, the solution at each grid point is obtained by backward time integration along a Lagrangian trajectory to locate the departure point and interpolation of the value at the departure point among the grid points. The semi-Lagrangian treatment of the advection equations allows a large time step and is unconditionally stable as the Lagrangian method. For convection–diffusion type

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equations it also produces a symmetric matrix and maintains the efficiency of symmetric solvers as the Eulerian method. Recent studies on the semi-Lagrangian method for the Navier–Stokes equations can be found in [19,20,9]. In [13,14,7,6], the semi-Lagrangian method is used to solve the level set equations. However, very few studies in the literature have discussed the potential of using the semi-Lagrangian method to achieve large time step VOF advection. A segment Lagrangian VOF method was developed in [4] for the 3D two-phase interfacial flows. It was claimed that CFL numbers greater than one could be applied, but no such tests were demonstrated in their paper. The difficulty to solve the VOF equations using the semi-Lagrangian scheme is that the VOF function is not continuous at the interfacial cells, and the interpolation schemes, such as the simple linear interpolation, spline interpolation, cubic interpolation propagation (CIP) [16], and high order ENO/WENO schemes [13,9], devised for the momentum and level set equations, are not suitable for the VOF function.

In order to eliminate the time step constraint due to the CFL condition for two-phase interfacial flows, a semi-Lagrangian, operator-splitting, high-performance volume-of-fluid advection scheme (SLOSH-VOF) is developed in the present study. Backward time integration using a second order midpoint method is performed for each cell face instead of the cell center with an operator splitting strategy to separately locate the departure cell in each spatial direction. The VOF values are reconstructed using a geometrically based interpolation algorithm where a PLIC scheme [8] is employed to calculate the volume fractions in the interfacial cells. The proposed method is simple, efficient and its implementation is easy. The performance of the semi-Lagrangian VOF method is evaluated through several benchmark advection tests. Compared to the Eulerian advection method, the time steps can be significantly increased and the computations can be substantially speeded up with the unconditionally stable property. The computed results using the SLOSH-VOF method match those obtained using the Eulerian method even with much larger CFL numbers (up to one order of magnitude), whereas the mass errors are greatly reduced by one to two orders of magnitude.

2. Numerical methods

2.1. Volume-of-fluid method

In the VOF method, the interface between liquid and gas is tracked by the VOF function, which is defined as the liquid volume fraction in a computational cell. The value of the VOF function in a cell is defined by

$$F(\mathbf{x},t)=\begin{cases}1, & \text{in the liquid},\\ 0<F<1, & \text{at the interface},\\ 0, & \text{in the gas}.\end{cases} \tag{1}$$

The position of the interface can be updated by solving the following VOF advection equation described using the Eulerian method,

$$\frac{DF}{Dt}\equiv\frac{\partial F}{\partial t}+\mathbf{u}\cdot\nabla F=0. \tag{2}$$

It should be noted that since the VOF function is not smoothly distributed at the interface, an interface reconstruction procedure is required to evaluate the VOF fluxes across a surface cell. The reconstructed interface can be propagated using either an Eulerian or a Lagrangian propagation scheme as implemented in some standard VOF methods [10,8,11,1,2,17]. The Lagrangian approach in these methods is used to propagate the *reconstructed interface* for evaluating volume fluxes across the cell faces, rather than the VOF function itself. Therefore, these advection schemes are still subjected to the CFL condition in order to satisfy the physical constraint on the volume fraction $0 < F < 1$ [8]. For clarity, the VOF methods using a Lagrangian approach to propagate the *reconstructed interface* will still be referred to as Eulerian methods in the present study.

2.2. Semi-Lagrangian advection scheme

The VOF advection equation Eq. (2) can be re-written in the Lagrangian form as

$$\frac{dF}{dt}=0, \tag{3}$$

$$\frac{d\mathbf{x}}{dt}=\mathbf{u}(\mathbf{x},t). \tag{4}$$

In the Lagrangian method, Eq. (3) is solved along the characteristic line defined using Eq. (4). In the semi-Lagrangian method, the VOF value, F^{n+1}, defined at each grid point is obtained by tracking backward to the previous time step along the characteristic line. Eqs. (3) and (4) can be discretized using the semi-Lagrangian scheme

$$\frac{F^{n+1}-F_d^n}{\Delta t}=0, \tag{5}$$

$$\frac{\mathbf{x}^{n+1}-\mathbf{x}_d^n}{\Delta t}=\mathbf{u}(\mathbf{x},t), \tag{6}$$