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Crack reconstruction using a level-set strategy

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ABSTRACT

We present a level-set based technique to recover key characteristics of a defect or crack (e.g. location, length and shape) in a two-dimensional material from boundary electrical measurements. The key feature of this work is to extend the usual level-set technique for modeling volumetric objects to very thin objects. Two level-set functions are employed: the first one models the location and form of the crack, and the second one models its length and connectivity. An efficient gradient based method is derived in order to define evolution laws for these two level-set functions which minimize the least squares data misfit. Numerical experiments show the utility of this method even in the presence of a significant noise level in the measurements. A finite element method is used to simulate the electric field behavior in the presence of very thin objects.

1. Introduction

The problem of determining the location, size and shape of flaws within a material from measurements on its surface is of much interest in non-destructive testing. This is an important problem from the computational, medical and industrial viewpoints. Boundary data can be obtained from thermal, acoustic, elastostatic, or electrostatic measurements, amongst others. For example, Friedman and Vogelius [22] initiated these studies for a steady state thermal problem and proved uniqueness for a single buried crack under some conditions; Liepa et al. [30] used electrical measurements to determine the location and size of a straight-line crack; Kress [27] considered the time-harmonic acoustic inverse problem for a sound-soft crack (or perfectly conducting) in $\mathbb{R}^2$. He extended this work also to the case of elastic wave scattering from a thin infinitely long cylindrical crack in [28].

As a model problem, we consider here a two-dimensional material Ω in which electric currents are created by applying voltage potentials γ_j at its boundary. By doing so, the electric potential u_j satisfies

$$\nabla \cdot b(\mathbf{x})\nabla u_j = 0 \quad \text{in } \Omega \tag{1}$$

and

$$u_j = \gamma_j \quad \text{on } \partial\Omega. \tag{2}$$

The voltages are applied at different points at the boundary in order to create sufficient diversity in the data.

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We are interested in the case where the conductivity profile $b(\mathbf{x})$ contains sharp discontinuities between the background material and a crack-like shape. Without loss of generality, we consider in this paper thin insulating cracks with a finite contrast in conductivity between the interior and the exterior of the crack, i.e., we model the cracks as very thin structures with a small and fixed thickness along the crack. The location, form and connectivity structure of the cracks is unknown, which means that the crack could consist of two (or more) different parts which are disconnected. The electrical properties in the whole domain are piecewise constant with only two possible values (referring to the interior and the exterior of the crack) which are known a priori. We assume that the electrical properties inside the crack region are sufficiently different from the background material. Therefore, our problem is an inverse crack problem related to electrical impedance tomography [24,8,15,9].

Regardless of the kind of measurements taken at the boundary, many different inversion approaches have been applied in the literature so far to detect the presence of a crack and to determine its location and size. Overall, we can distinguish two classes of methods: non-iterative methods and iterative methods.

Non-iterative methods, such as the reciprocity gap technique for straight-line cracks [3,2] or the factorization method [10], are fast but generally do not provide detailed information about the crack. Baratchart et al. [5] introduced a meromorphic approximation approach to solve some 2D inverse problems for the Laplacian. Even though this method only requires one numerical experiment, it can only locate the end points of the crack.

On the other hand, iterative methods are computationally more expensive. They need to solve a direct problem at each step to compute the Fréchet derivative of the underlying forward operator, but they usually offer very accurate results (provided that a good initial estimate of the crack is used). These methods have been studied extensively over the last two decades. Among these methods, we mention [27,28] where the crack is found from the measured far-field data using a regularized Newton method. Recently, Kress and Serranho [29] proposed a hybrid method to solve the inverse scattering problem for sound-soft cracks in two dimensions. In that work, the authors combine ideas of both, non-iterative and iterative methods. We also want to mention here that sampling methods have been applied to the inverse crack problem [26,13] with success.

For more information regarding the very interesting existing literature on crack detection problems, we refer the reader to consulting the expositions given in [6,7,14,19,32,38], and the further references given there. We also want to mention the recent thesis [36] which presents and compares various techniques for crack detection in non-destructive testing and the related work in [1] which treats cracks in 3D domains by a volumetric level set approach for eddy current imaging.

The main purpose of this paper is to investigate the potential of shape-based reconstruction algorithms that use level-set techniques to recover the location and geometry of a disconnected snake-like crack. Since the seminal work in [39], level-set techniques (originally introduced in [34] for computational front propagation) have been widely used for solving inverse problems with great success [39,31,21,17,12,25,37] (see [18] for a recent overview on level-set techniques applied to inverse problems). However, level-set techniques are intrinsically designed for the case of volumetric objects. If these objects have negligible volume, as is commonly the case in flaws encountered in materials, the standard level-set technique cannot be applied in a straightforward manner. It is our goal here to present one possible extension of the level-set technique for the inverse crack problem. By some appropriate adaptations of classical tools to this new situation, we give evolution laws for an initial guess in both its normal and tangential directions during the iterative process. The evolution of the normal component will correspond to the movement of the crack over the domain, whereas a tangential evolution will affect the length and connectivity of the crack. We present an efficient two-step numerical scheme which during the early iterations intends to reduce the data misfit by considering an evolving straight-line (multi) crack, and during the later iterations intends to find more details of the crack, in particular its curvature at each point. Our technique relies on the assumption that the nonlinear cost functional representing the data misfit can be approximated locally by a linear operator.

We mention that the direct problem of moving a curve using a level-set approach has already been studied by Burchard et al. [11], amongst others. They represent the evolving curve in R^3 by the intersection between the zero level sets of two level-set functions, and they show that their representation automatically handles mergings and breakings of the curve under a variety of geometrically based motions. The motion of curves in R^2 and R^3 is important to model several physical phenomena such as crack growth [42–44,16]. In [42,43], the authors present an algorithm which couples the level-set method with the extended finite element method for modeling propagating cracks. Instead, an element-free Galerkin method is used in [44] with a vector level-set method. Also, a numerical method that couples the extended finite element method to the fast marching method has been proposed in [16]. A review of the different techniques for tracking the motion of a curve by level-set functions can be found in [20].

The paper is organized as follows. In Section 2 we explain our novel approach for representing cracks by a pair of two level-set functions. In Section 3 we state the inverse problem and present the numerical algorithm for the reconstruction of the shapes. In Section 4 we give some technical details about the numerical method, the type of measurements we use, the regularization applied to stabilize the reconstruction process, and other details about the reconstruction algorithm. In Section 5 we show several numerical experiments which demonstrate the performance of our new algorithm. Section 6 contains our conclusions and some hints to future work.

2. Modeling cracks using level-sets

As we have mentioned in the introduction, the current paper is an attempt towards using level-set techniques for reconstructing objects with negligible volume. Our objective is to reconstruct the thin objects (including their connectivity) using