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An all-speed relaxation scheme for interface flows with surface tension

Benjamin Braconnier [a,c,*], Boniface Nkonga [b,c]

[a] Glaizer Group: Générateur de Technologies Innovantes, 32 rue Guy Moquet, 92240 Malakoff, France
[b] Laboratoire J.A. Dieudonné, U.M.R. C.N.R.S. No. 6621, Université de Nice Sophia-Antipolis, Parc Valrose, 06108 Nice Cedex 02, France
[c] MAB, Univ. Bordeaux 1/INRIA Futurs, ScAlApplix 351 cours de la libération, 33400 Talence, France

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ABSTRACT

We consider interface flows where compressibility and capillary forces (surface tension) are significant. These flows are described by a non-conservative, unconditionally hyperbolic multiphase model. The numerical approximation is based on finite-volume method for unstructured grids. At the discrete level, the surface tension is approximated by a volume force (CSF formulation). The interface physical properties are recovered by designing an appropriate linearized Riemann solver (Relaxation scheme) that prevents spurious oscillations near material interfaces. For low-speed flows, a preconditioning linearization is proposed and the low Mach asymptotic is formally recovered. Numerical computations, for a bubble equilibrium, converge to the required Laplace law and the dynamic of a drop, falling under gravity, is in agreement with experimental observations.

0. Introduction

Modeling and computing multi-fluid flows is a very active research area. This is due to the fact that multi-fluid flows play fundamental roles in many natural phenomena used in industrial process. The diversity of the characteristic scales in one phenomena leads to large difficulties when numerical simulation is considered. Although the constant evolution of computational resources enables finest resolutions, numerical methods accuracy and efficiency always need to be improved.

There are two main difficulties specific to interfaces flows: the interface shape dynamic and the wave transmission. *Interface tracking* approaches use an explicit definition of the interface. The *front tracking method* [17] uses a lower dimensional grid to represent interfaces in numerical solutions. Propagation of the front is obtained by the requirement of jumps conditions. For the *Volume of Fluid* [38] method, the interface is described by an average fluid fraction function and the interface is reconstructed by different approaches. The main difficulties of these methods are related to shape topology changes and they are not well adapted for multiple interfaces flows such as sprays. The *level set* method [15] overcomes these difficulties by embedding interfaces in a distance function. Unfortunately, although many recent works deal with accurate wave transmission (GFM [15], SFM [2]), multidimensional and unstructured grid extension needs to be improved [32]. An other way to overcome the difficulties related to sharp interfaces is to artificially smooth the interface (*diffuse interface methods*) and create a mixture zone such as to recover the physical properties of the sharp interface. This leads generally to non-conservative hyperbolic models [5,29] for which the weak solutions of finite volume approximations are not well defined. Nevertheless, the relevance of this method has been successfully proved in [37] for inviscid high speed flows. Surface tension

* Corresponding author. Address: Glaizer, 32 rue Guy Moquet, 92240 Malakoff, France.
E-mail addresses: benjamin.braconnier@glaizer.com (B. Braconnier), boniface.nkonga@unice.fr (B. Nkonga).

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forces has been considered in [35] and the low mach preconditioning in [20]. For theses works, the discrete non-conservative operators are obtained by the inversion of well balanced relations [1].

In the context of diffuse interface method, this paper focus on computations of low Mach interface flows submitted to surface tensions forces. The physical model is based on the five equation model [29,33], a simplified version of the Baer Nunziato model [5]. As the interface is not sharply defined, the capillary effects are formulated by a *Continuum Surface Force* (CSF [9]) which uses a color function. The finite volume approximation is formulated in terms of local fluctuations deduced from the approximate resolution of Riemann problems. Indeed, in order to reduce the currently observed parasitic currents [27], the surface tension terms needs to be considered as part of the non-conservative hyperbolic subsystem. This non-conservative formulation must be preserved [16,30] in order to avoid wrong discontinuous solutions [24]. Its resolution needs the definition of regularizing viscous paths [16] or the elaboration of kinetic relations [23]. Despite, when the surface tension forces model relies on a color function constant across shock and rarefaction waves, surface tension terms can be reformulated in a conservative form [18]. In [35], a numerical scheme based on a Godunov method is proposed for this specific case. It formally and numerically recovers the Laplace law and enable the computation of complex interface flows. Unfortunately, it can not be easily generalized to the system [29] which volume fraction equation is not a simple transport equation or when complex equations of state are considered. In [3,33], balance equations between conservative and non-conservative terms are pointed out in order to derive consistent discrete non-conservative operators [1]. The major drawback of this method is its complexity when the model include too many non-conservative operators. In our work, we propose an approximate Riemann solvers based on the relaxation scheme [12,28,31,39,14] which yields an implicit viscous regularization of non-conservative operators. Thus, fluctuations are evaluated by the virtue of a linearized non-conservative Riemann problem that takes into account surface tension force as a first order derivative. Their expressions are analytically formulated and without thermodynamical parameters. The resulting scheme is thus very efficient. Moreover, considering asymptotic expansions, we formally prove that the non-conservative operator regularization is consistent with the mathematical model. Then, using the pressure as an independent variable in the relaxation scheme gives a natural way to propose a low Mach preconditioning by the introduction of the times scales related to the acoustic and the material waves.

The paper is organized as follows. We describe, in Section 1, the governing equations. In Section 2, the finite volume approximation is defined. A relaxation system is proposed for the physical model and the solution of the associated Riemann problem is constructed. The main numerical scheme is then achieved. Section 3 is devoted to the presentation of some specific numerical tools: low Mach preconditioning, surface tension approximation and implicit schemes. In Section 4, we discuss the numerical results obtained for a bubble at equilibrium under the Laplace law, shock bubble interactions and a falling water drop. In the last section, we propose discussions of the achievement and draw inferences for further work.

1. Mathematical model

Let us consider a multi-phase flow and use the subscript k for the specification of each component. The physical model considered in this paper assumes that pressures and velocities are at equilibrium. The physical model, obtained as an asymptotic limit of the non-equilibrium Baer–Nunziato system [5], is given by a single pressure and velocity multi-phase flow [29,19]:

$$\begin{cases} \partial_t(\alpha_k) + \mathbf{u}\cdot\nabla(\alpha_k) - \beta_k\nabla\cdot(\mathbf{u}) = 0, \\ \partial_t(\alpha_k\rho_k) + \nabla\cdot(\alpha_k\rho_k\mathbf{u}) = 0, \\ \partial_t(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u} + p) = \nabla\cdot(\tau) + \mathbf{f}_S + \rho\mathbf{g}, \\ \partial_t(E) + \nabla\cdot(\rho H\mathbf{u}) = \nabla\cdot(\tau\mathbf{u}) + \mathbf{u}\cdot(\mathbf{f}_S + \rho\mathbf{g}) - \nabla\cdot(\mathbf{q}), \end{cases} \tag{1}$$

where ρ is the mixture density, $\mathbf{u}$ is the fluid velocity, E the total energy, p the pressure and H the enthalpy. For each component, α_k is the volume fraction, ρ_k is the partial density, $\varepsilon_k = \varepsilon_k(\rho_k, p)$ is the partial internal energy. The mixture density, internal energy, total energy and the enthalpy are defined as:

$$\rho = \sum_k \alpha_k\rho_k, \quad \rho\varepsilon = \sum_k \alpha_k\rho_k\varepsilon_k, \quad E = \rho\varepsilon + \frac{1}{2}\rho\mathbf{u}\cdot\mathbf{u}, \quad H = \frac{E+p}{\rho}.$$

We assume that the equation of state associated to each component is governed by the stiffened gas EOS: $p(\rho_k, \varepsilon_k) = (\gamma_k - 1)(\rho_k\varepsilon_k - \rho_k\varepsilon_k^\infty) - \gamma_k p_k^\infty$, where ε_k^∞ and p_k^∞ are fixed reference thermodynamic states. The perfect gas EOS is recovered when $\varepsilon_k^\infty = p_k^\infty = 0$. According to the pressure equilibrium and the constraint of entropy conservation along phase trajectories [19], the pressure p is a combination of the equation of state associated to the components of the flow:

$$\frac{p + \gamma p^\infty}{\gamma - 1} = \rho(\varepsilon - \varepsilon^\infty), \tag{2}$$

where the mixture parameters γ, p^∞ and ε^∞ depends on the volume fraction and are defined by the relations:

$$\frac{1}{\gamma - 1} = \sum_k \frac{\alpha_k}{\gamma_k - 1}, \quad \frac{\gamma p^\infty}{\gamma - 1} = \sum_k \frac{\alpha_k\gamma_k p_k^\infty}{\gamma_k - 1} \quad \text{and} \quad \rho\varepsilon^\infty = \sum_{k=1}^{q} \alpha_k\rho_k\varepsilon_k^\infty. \tag{3}$$