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On thin gaps between rigid bodies two-way coupled to incompressible flow

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ABSTRACT

Two-way solid fluid coupling techniques typically calculate fluid pressure forces that in turn drive the solid motion. However, when solids are in close proximity (e.g. touching or in contact), the fluid in the thin gap region between the solids is difficult to resolve with a background fluid grid. Although one might attempt to address this difficulty using an adaptive, body-fitted, or ALE fluid grid, the size of the fluid cells can shrink to zero as the bodies collide. The inability to apply pressure forces in a thin lubricated gap tends to make the solids stick together, since collision forces stop interpenetration but vanish when the solids are separating leaving the fluid pressure forces on the surface of the solid unbalanced in regard to the gap region. We address this problem by adding pressure degrees of freedom onto surfaces of rigid bodies, and subsequently using the resulting pressure forces to provide solid fluid coupling in the thin gap region. These pressure degrees of freedom readily resolve the tangential flow along the solid surface inside the gap and are two-way coupled to the pressure degrees of freedom on the grid allowing the fluid to freely flow into and out of the gap region. The two-way coupled system is formulated as a symmetric positive-definite matrix which is solved using the preconditioned conjugate gradient method. Additionally, we provide a mechanism for advecting tangential velocities on solid surfaces in the gap region by extending semi-Lagrangian advection onto a curved surface mesh where a codimension-one velocity field tangential to the surface is defined. We demonstrate the convergence of our method on a number of examples, such as underwater rigid body separation and collision in both two and three spatial dimensions where typical methods do not converge. Finally, we demonstrate that our method not only works for the aforementioned "wet" contact, but also works in conjunction with "dry" contact where there is no fluid in the gap between the solids.

1. Introduction

Solid fluid coupling is important in many areas of science and engineering such as ship and aircraft design, the study of aneurysms and heart valves, etc. Numerical methods for modeling two-way solid fluid coupling can be loosely classified into two categories: partitioned and monolithic. Partitioned methods typically separately evolve the fluid and solids using the results of one as boundary conditions for the other in an alternating one-way coupled fashion, see e.g. [30,42,11]. Multiple iterations are typically required for stability, although there is no guarantee that iterations converge at all often forcing one to take excessively small time steps. Partitioned approaches can also suffer from other problems such as the

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added-mass instability, see e.g. [4]. The main advantage of partitioned methods revolves around the ability to reuse existing code targeted to either solids-only or fluids-only problems. Monolithic methods aim to more fully two-way couple the fluid and solids alleviating a number of the aforementioned issues, see e.g. [25,20,2,27,33,12].

Our method is based on the method in [33] which evolves the explicitly treated components of both the fluid and solids independently and solves for the fluid pressures and the solid velocities together in a single monolithic two-way coupled system during the projection step. Whenever a line segment connecting two adjacent grid cell centers intersects a solid surface, a solid fluid coupling equal velocity constraint is enforced on the common grid face as long as at least one of the neighboring cells is a fluid cell. An interpolation operator is defined to interpolate solid velocities from solid surfaces to these constrained grid faces, and the transpose of this interpolation operator conservatively distributes fluid pressure impulses from these grid faces back to the solid surfaces. The gradient and divergence operators are discretized to be the negated transpose of each other resulting in a symmetric linear system, which can be made positive-definite in the deformable body case by factoring the damping matrix allowing for the use of fast linear solvers such as preconditioned conjugate gradients.

Although the method of [33] has been demonstrated to be a suitable approach for two-way coupling both rigid and deformable bodies (both volumetric and thin shell) with an incompressible fluid, we have observed erroneous results when solid bodies are in close proximity and multiple solid boundaries are rasterized to a single grid face. For example, when two rigid bodies are in contact, there will be grid faces in the gap between the two solids whose two adjacent grid cell centers are rasterized to different solids. The method of [33] does not assign solid fluid coupling constraints to such grid faces, since cells on the two sides of such a grid face are both inside solids. Thus, fluid pressure forces are absent inside this gap region producing nonphysical behavior. Although collision and contact stops the solids from interpenetrating, these inequality constraints vanish when the solids are separating leaving only an unbalanced fluid pressure force to spuriously push the solids back together. Moreover, in order for the solids to separate, fluid must flow into the thin gap region. Thus it is also important to model tangential flow in the gap region, incorporate the resulting velocity flux into the velocity divergence, and allow such flows to couple with the exterior flow on the grid.

It is instructive to consider how other approaches behave in the scenario where two solids come into contact with each other. One could use adaptive and/or moving body-fitted grids in order to increase the grid resolution in the gap region, for example solving with an Arbitrary Lagrangian–Eulerian (ALE) method, see e.g. [19,10]. However, as the solids approach and eventually touch, the grid resolution required for resolving the gap region will become infinite in finite time. As an interesting alternative to body-fitted grids, methods such as those proposed in [9,26] use a fixed-size Cartesian grid modifying the stencil near the solid boundary. In such methods, grid cell sizes will not approach zero as the solids come together because boundary conditions are enforced using ghost cells inside the solids. However, the ghost cells only work when there are real fluid cells for them to interact with, and all the real fluid cells disappear when solids come into close contact. Similarly, methods such as [3,17,18,36] which rely on cut cells that are merged with full cells to avoid accuracy and time step restrictions also cannot be applied when all the full cells vanish as the solids come into close contact. Fictitious domain (see e.g. [13]) and immersed boundary methods (see e.g. [28,29] and the references therein) intrinsically avoid the problem of vanishing or disappearing fluid grid cells by discretizing the fluid on every grid cell whether it is inside the solid or not. The fictitious domain method has an incompressibility constraint in the fluid region and a solidification constraint for the grid cells which are determined to be in the solid region. If two solids are in close proximity and both the incompressibility constraint and the solidification constraint are enforced precisely, the method lacks the degrees of freedom to represent the incompressible flow that allows fluid to flow into the gaps as the solids separate. On the other hand, relaxing the constraints even via numerical means could allow degrees of freedom between the two solids to become unsolidified and instead represent tangential fluid flow in the gap. However, this gives a much less precise description of the solid and may produce artifacts of its own. The immersed boundary method uses a smoothed out approximation of the Dirac delta function in order to define a forcing back and forth between the fluid and the solid, inherently more loosely enforcing the solidification constraint than using the fictitious domain method. Therefore, the immersed boundary method more readily allows the degrees of freedom between the two solids to model the necessary tangential flow allowing bodies to separate while immersed in the incompressible fluid. Accuracy on the other hand is another matter. This tangential flow will compete with the smeared out region where the immersed boundary method is attempting to constrain the motion of the underlying fluid degrees of freedom to follow the solid motion. This lack of accuracy can cause unwanted stress in the solid structure, and attempts to minimize this stress can stop the unwanted deformation but also stop the desired tangential flow.

In the framework of [33], we propose adding fluid pressure degrees of freedom to solid surfaces in order to provide fluid pressure forces in thin lubricated gap regions. These additional fluid pressure degrees of freedom are added to the particles of solid surface meshes which can be readily refined or coarsened according to the resolution needed for resolving the fluid flow in the gap. We only deal with volumetric rigid bodies in this paper and rasterize each rigid body as a closed hull consisting of both grid faces between the fluid and solids as well as grid faces between two different solids. We treat each grid face between two solids, where solid fluid coupling constraints are missing in [33], as two virtual faces sandwiching a virtual fluid cell in between. Additional solid fluid coupling constraints are added to these virtual faces, and the fluid pressure forces on these faces are computed using an interpolation operator that interpolates fluid pressures from solid surface pressure degree of freedom particles to virtual fluid cells. The transpose of this interpolation operator is then used to conservatively distribute the velocity flux through these virtual faces to pressure degree of freedom particles for