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Uneven-order decentered Shapiro filters for boundary filtering

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ABSTRACT

This paper addresses the use of Shapiro filters for boundary filtering. A new class of uneven-order decentered Shapiro filters is proposed and compared to classical Shapiro filters and even-order decentered Shapiro filters. The theoretical analysis shows that the proposed boundary filters are more accurate than the centered Shapiro filters and more robust than the even-order decentered boundary filters usable at the same distance to the boundary. The benefit of the new boundary filters is assessed for computations using the compressible Euler equations.

1. Introduction

Low-pass filters are used for filtering high-frequency waves in many numerical simulations such as weather forecasting [1], computational fluid dynamics [2] and computational aeroacoustics [3]. Simple and efficient high-order filters that preserve the largest structures and fully damp the 2Δ-waves were proposed by Shapiro [4–6], who introduced a class of explicit linear filters of $2n$-order accuracy, using a stencil of $(2n+1)$ points. This approach has been pursued in the research fields mentioned above and led to a wide range of linear filters with variable cut-off frequencies and accuracy orders, whether explicit [7–10] or implicit in space [11–17], all of them intrinsically based on Shapiro filters and referred to as generalized Shapiro filters [18]. Although many numerical simulations are performed in finite area, few filters have been developed specifically for boundary filtering, some exceptions being the decentered implicit and explicit generalized Shapiro filters proposed in Refs. [19] and [20]. Both these studies show that the decentered filters must be carefully tuned to possess good accuracy, damping and dispersion properties.

The present work addresses the problem of boundary filtering using high-order decentered Shapiro filters. The generic formulas and properties of the $2n$-order centered Shapiro filters are summarized in Section 2. These formulas are extended to the $2n$-order decentered Shapiro filters in Section 3 which discusses also the inherent anti-dissipative nature of such decentered filters. A new class of $(2n+1)$-order decentered Shapiro filters with good accuracy, damping and dispersion properties is proposed in Section 4 and compared with $2n$-order centered and decentered Shapiro filters for numerical applications based on the compressible Euler equations in Section 5.

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Table 1
Binomial coefficients of $(a-b)^n$ and successive compositions of the standard finite difference operator δ.

	w_{i-3}	$w_{i-\frac{5}{2}}$	w_{i-2}	$w_{i-\frac{3}{2}}$	w_{i-1}	$w_{i-\frac{1}{2}}$	w_i	$w_{i+\frac{1}{2}}$	w_{i+1}	$w_{i+\frac{3}{2}}$	w_{i+2}	$w_{i+\frac{5}{2}}$	w_{i+3}
$\delta^0 w_i$							1						
$\delta^1 w_i$						−1		1					
$\delta^2 w_i$					1		−2		1				
$\delta^3 w_i$				−1		3		−3		1			
$\delta^4 w_i$			1		−4		6		−4		1		
$\delta^5 w_i$		− 1		5		−10		10		− 5		1	
$\delta^6 w_i$	1		−6		15		−20		15		−6		1

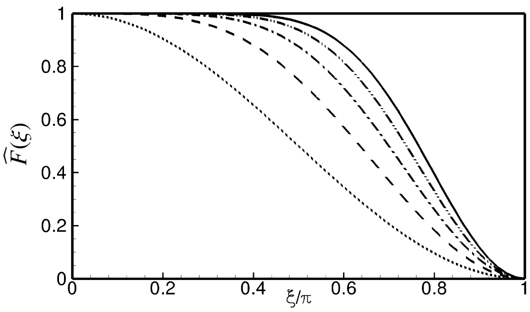

Fig. 1. Response functions of the 2nd (▪▪▪▪), 4th (▬ ▬ ▬), 6th (▬ ▪ ▬), 8th (▬ ▪ ▪ ▪ ▬) and 10th (▬▬▬) order Shapiro filters.

2. Shapiro filters

Shapiro filters [4–6] are one-dimensional linear symmetrical operators that remove 2Δ-waves completely while preserving longer wave components. Considering a uniform grid ($x_i = i\Delta x$) with spatial step Δx, the filtered value $F^{2n}(w_i)$ of a quantity w_i at point x_i using a $2n$-order Shapiro filter reads:

$$F^{2n}(w_i) = w_i + (-1)^{n-1}\frac{\delta^{2n}}{2^{2n}} w_i \tag{1}$$

where n is a positive integer and δ^{2n} denotes $2n$ successive applications of the standard difference operator [21–23] defined by $\delta w_i = w_{i+\frac{1}{2}} - w_{i-\frac{1}{2}}$. The coefficients applied to the discrete values of w for n successive applications of the δ operator correspond to the binomial coefficients of $(a-b)^n$, see Table 1. The Taylor expansion of formula (1) yields:

$$F^{2n}(w) = w + (-1)^{n-1}\frac{\Delta x^{2n}}{2^{2n}}\frac{\partial^{2n} w}{\partial x^{2n}} + \mathcal{O}\left(\Delta x^{2n+2}\right) \tag{2}$$

so that Shapiro filters are consistent with the identity operator plus a high-order linear diffusion [5] of order $2n$ with a coefficient $K_{2n} = (-1)^{n-1}\Delta x^{2n}/2^{2n}$. Introducing the reduced wave number $\xi = k\Delta x$, the response function $\widehat{F}^{2n}$, or Fourier symbol, of a Shapiro filter is:

$$\widehat{F}^{2n}(\xi) = 1 - \sin^{2n}\left(\frac{\xi}{2}\right) \tag{3}$$

As shown in Fig. 1, Shapiro filters become more selective as the accuracy order increases. Shapiro filters are the cornerstone [18] of many filters developed for weather forecasting [1,8,14], high-order computational fluid dynamics [2,15,17,24] and computational aeroacoustics [3,9,10].

3. Shapiro filters and boundary filtering

Several techniques can be applied to filter a quantity near the boundary of the computational domain. The simplest method, from a filtering point of view, consists in creating as many ghost points as necessary to use the standard centered stencil of the filter. The difficulty is then reported on the values to be applied in the ghost cells [4]. The second method, initially proposed in [25], consists in decreasing locally the accuracy of the filter to apply low-order centered formulas near the boundaries. This technique is very robust and easy to implement since it does not require higher-order boundary conditions [26]. The drawback is that it increases locally the dissipation which can lessen the global accuracy of the simulation if