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A high resolution differential filter for large eddy simulation: Toward explicit filtering on unstructured grids

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ABSTRACT

A high resolution, low-pass differential filter for numerical simulations on unstructured grids is proposed. The finite element discretization of the filter equation on a structured grid results in a discrete compact filter. The proposed filter is significantly less dissipative than Germano's differential filter, while completely suppresses fluctuations at the grid cut-off frequency. Manufactured solutions were used to verify the performance of the proposed filter. The results suggest that the proposed filter will be very effective for explicit filtering and approximate deconvolution modeling in Large Eddy Simulations.

1. Introduction

Large Eddy Simulation (LES) of turbulent flows has found widespread applications both in academia and industry. The idea behind Large Eddy Simulations (LES) is to decompose the flow properties into a large-scale or resolved component, $\bar{\phi}$, and a small-scale or subgrid component, ϕ_{sg} [1]. This decomposition is achieved by applying a low pass, spatial filter. The filter width usually corresponds to the smallest scale resolved on the grid.

It is customary to *implicitly* apply the filter by solving the filtered Navier–Stokes equations with a presumed subgrid-scale stress model. This procedure is known as *implicit filtering* [2]. In implicit filtering, the filter function is not necessarily known as it is implicitly defined by the subgrid model and the numerical grid [2–4]. Implicit filters, therefore, do not allow the control of numerical errors caused by truncation errors and aliasing of high frequencies [5]. Moreover, the filter spectral distribution and its energy dissipation cannot be quantified [2,6,7].

These problems can be addressed by using LES methodologies based on *explicit filtering* of the flow field, namely the Approximate Deconvolution Model (ADM) [7–10], and the relaxation filtering (RF) technique [11–16]. In these methods, filtering is an explicit part of the numerical simulation to prevent the energy accumulation at the grid cut-off. Since the filter operator is known, the energy dissipation associated with filtering can be quantified. So far, successful applications of these methods have only been reported on structured grids, for which a discrete high-order filter operator can be easily constructed. Examples include the use of Lele's compact filters [17] by Visbal and Rizetta [18], Rizzeta et al. [19], Uzun and Hussaini [20,21], as well as explicit discrete filters by Bogey and Bailly [12,13], Berland et al. [15], and Fauconnier et al. [16].

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Nomenclature

h	Local filter width
$\mathbf{J}$	Jacobian matrix
k	Magnitude of wavenumber
$\mathbf{k}$	Three dimensional wavenumber vector
m_{ij}	Component of filter left-hand-side mass matrix
n_{ij}	Component of filter right-hand-side mass matrix
N_j	Shape function for local node j
$T(\mathbf{k})$	Filter transfer function
x, y	Spatial coordinates in the physical space

Greek

α	Filter parameter for filter cut-off frequency in the computational space
α_{ij}	Components of filter cut-off frequency parameter in the physical space
α_f	Filter parameter for compact notation
β	Filter parameter for grid cut-off frequency in the computational space
β_{ij}	Components of grid cut-off frequency parameter in the physical space
δ	Germano's filter radius
ϕ	Unfiltered scalar variable
$\overline{\phi}$	Filtered scalar variable
ξ, η	Spatial coordinates in the two-dimensional computational space
Ω	Arbitrary element in the physical space
$\widetilde{\Omega}$	Transformed element in the computational space, a.k.a. reference element

Subscript

cnt	Continuous
fc	Filter cut-off
g	Grid cut-off
Gr	Germano's

The extension of discrete filter operators to unstructured grids is not straightforward which has hampered the use of ADM and RF for LES on unstructured grids. Marsden et al. [22] and Haselbacher and Vasilyev [23] suggested explicit filtering procedures for unstructured grids based on a weighted sum of neighbouring node values. Both of these methods have drawbacks which have hampered their application. In particular, it is not possible to ensure the stability of the filter operator (i.e. $G(k) \le 1, \forall k$) in a general mesh topology. Moreover, the spectral distribution of the filter kernel is strongly dependent on the distribution of surrounding nodes. The filter of Marsden et al. [22] also requires the careful selection of a subset of neighbouring nodes which might not exist in the presence of skewed and stretched elements [24].

Another approach for the design of discrete filter kernels is the use of differential operators. First introduced in the 1980's by Germano [25,26], differential filters for LES applications have been used by Mullen and Fischer [27], You et al. [28], and Bose et al. [24] Germano's differential filter is defined by the following differential equation:

$$\overline{\phi} - \delta^2 \frac{\partial^2 \overline{\phi}}{\partial x_j \partial x_j} = \phi \,, \tag{1}$$

where ϕ and $\overline{\phi}$ are unfiltered and filtered variables, respectively, and the filtering parameter, δ, controls the filter's attenuation. Germano's filter transfer function in the Fourier domain is given by

$$T_{Gr}(k) = \frac{1}{1 + \delta^2 k^2} \,, \tag{2}$$

where k is the wavenumber. Germano's filter is stable (i.e. $|T_{\mathrm{Gr}}| \le 1$) for all values of wavenumber, and is a useful tool for theoretical analysis [29]. However, Germano's filter has some limitations for practical applications in LES. In particular, its transfer function, Eq. (2), never reaches zero in the discrete form. Therefore, Germano's filter does not completely remove the energy content of the subgrid scales. This means that Germano's filter, if used without any additional dissipation or subgrid model, cannot prevent energy accumulation near the grid cut-off, which eventually destabilizes the numerical simulation. Moreover, Germano's filter is rather dissipative at low to moderate wavenumbers, corresponding to resolved scales.

The aforementioned shortcomings of Germano's filter are addressed by a new differential filter proposed in this paper. The transfer function of the filter is designed to remain close to unity over a wide range of wavenumbers, resulting in minimal dissipation over resolved scales, and to decline sharply to zero near the grid cut-off in order to completely remove the subgrid energy content.

The paper is organized as follows. A brief discussion of the definition of grid cut-off wavenumber is provided in Section 2. The new filter is introduced in Section 3. In Section 4, it is shown that a Galerkin discretization of the filter in one dimension results in the second order compact filter introduced by Lele [17]. The spectral accuracy of the proposed filter is compared with that of other conventionally used models in Section 5. The derivation of the filter for two-dimensional triangular elements is presented in Section 6. In Section 7, numerical examples are used to illustrate the characteristics of the proposed filter. Conclusions are drawn in Section 8.