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Inverse scattering problem from an impedance obstacle via two-steps method

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ABSTRACT

In this paper we deal with the inverse scattering problem for an impedance obstacle. The aim is to recover both the impedance function and the scatterer simultaneously. Based on boundary integral equations, our method splits the inverse problem into a well-posed direct problem followed by a smaller ill-posed problem which has advantages both in understanding the inverse problem and in the numerical reconstructions.

1. Introduction

The inverse problem aiming on the identification of the unknown objects or the investigation of the physical properties of the objects has received more and more attentions in the past decades not only because of its pure mathematical interest but also its applicability in the physical world. In this paper we consider an inverse scattering problem from an impedance obstacle. Our goal is the simultaneous reconstruction of the impedance function and the unknown obstacle from the measured scattered far field pattern. The impedance boundary conditions can be used to model practical problems like surface coating which has its application in antenna design, seismic exploration or medical imaging, for example.

The central tool for our approach is boundary integral equation which enables the transforming of a boundary value problem in the unbounded domain into a boundary integral equation. One advantage is immediately clear: the reduction of dimension by one. Roughly speaking, integral equations based methods for inverse scattering problems often lead to the following so-called far-field equation

$$\mathbf{F}(\partial D, \Pi) = u_\infty \tag{1}$$

where ∂D denotes the boundary of the scatterer and the vector Π contains quantities appear in the formulation of the problem or turn up in the solution process for the direct problem. Π depends hence both on the problem and on the solution method. For the inverse problem, it is therefore natural to attempt retrieving the information interested through this mechanism from the measured far-field pattern u_∞. In general, this seemingly simple equation however, can't be solved in any fashion because of the compactness of the operator $\mathbf{F}$. We will return to this point later. Another property of $\mathbf{F}$ is the nonlinearity w.r.t. ∂D which, on the other hand, can be handled by any linearization method such as Newton's method easily. Due to its simplicity, we will utilize Newton's method in our numerical realization and hence a short discussion of related researches from the literature is given here.

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In [10], an inverse impedance problem for recovering both the scatterer and the impedance is considered. Based on the classical Newton's method, a related direct problem must be firstly resolved at every iterative step. It is therefore time-consuming and complicated. Their approach has been modified and extended to the recovery of the impedance function for a given obstacle with phase-less far-field data in [7]. The inverse impedance problem for recovering the shape of the obstacle when the impedance function is explicitly given is studied in [12] based on nonlinear integral equations which avoids the computation of a related direct problem at each iteration. But the simultaneous determination of the obstacle and the boundary condition is not discussed therein.

The idea of our approach is to split the inverse problem into two steps in a natural way. One of the two parts is the well-posed direct problem and the other is a smaller ill-posed problem. The well-posed part can be solved with the direct solver and the ill-posed part can be dealt with the nonlinear integral equation method. One advantage is immediately clear: the size of the ill-posed problem is reduced.

The plan of this paper is as follows. In Section 2 the impedance problem will be posed and the solution methods for the direct problem will be briefly sketched. The two-steps method will be discussed in Section 3 in details. Several numerical reconstructions will be given in Section 4 to demonstrate our algorithm.

2. The impedance problem

The mathematical modeling for scattering of time-harmonic acoustic or electromagnetic waves from an infinitely long coated cylindrical objects with cross-section D in the plane leads to an impedance boundary value problem for the Helmholtz equation in the exterior domain of D. Assume $D \subset \mathbb{R}^2$ to be a C^2-domain, that is, the boundary ∂D allows an injective and two times differentiable parametrization $\gamma : [0, 2\pi] \to \mathbb{R}^2$. Given a planar incident wave $u^i(x) := e^{ik\langle x,d\rangle}$ with incident direction d for a wave number $k > 0$, we have the following

Problem 1 *(The direct impedance problem).* Find a solution $u^s \in C^2(\mathbb{R}^2 \setminus D)$, which can be continuously extended to $\overline{\mathbb{R}^2 \setminus D}$ and satisfies the following conditions:

1. $\Delta u^s + k^2 u^s = 0$ in $\mathbb{R}^2 \setminus D$.
2. The normal derivative

$$\frac{\partial u^s(x)}{\partial \nu} := \lim_{h\to 0^+} \big\langle \nu(x), \operatorname{grad} u^s\big(x + h\nu(x)\big)\big\rangle \tag{2}$$

exists for all $x \in \partial D$, where ν denotes the unit outward normal to ∂D.

3. For the real-valued function $\lambda \in C(\partial D)$, it holds the following impedance boundary conditions

$$\frac{\partial u}{\partial \nu} + \lambda u = 0 \quad \text{on } \partial D \tag{3}$$

where $u := u^s + u^i$ is the total field.

4. u^s satisfies the *Sommerfeld radiation condition*, i.e.

$$\lim_{r\to\infty} \sqrt{r}\left(\frac{\partial u^s}{\partial \nu} - iku^s\right) = 0, \quad r := |x|, \tag{4}$$

uniformly in all directions $\hat{x} := \frac{x}{|x|}$.

A solution to the direct problem is radiating and has an asymptotic behavior of the form

$$u^s(x) = \frac{e^{ik|x|}}{\sqrt{|x|}}\left\{u_\infty(\hat{x}) + O\left(\frac{1}{|x|}\right)\right\} \quad |x| \to \infty \tag{5}$$

uniformly for all directions $\hat{x} \in \Omega := \{x \in \mathbb{R}^2 \mid |x| = 1\}$. The function $u_\infty : \Omega \to \mathbb{C}$ is called the far-field pattern and describes the behavior of the scattered wave at the infinity.

Using potential method, this problem can be shown to be uniquely solvable through a combined potential ansatz (see [4], Sect. 3.7). For our purpose, we shall briefly describe it here. It is assumed that the solution to the direct problem is given by the combined potential ansatz

$$u^s(x) := \int_{\partial D} \frac{\partial \Phi(x, y)}{\partial \nu(y)} \varphi(y)ds(y) + \int_{\partial D} \Phi(x, y)\varphi(y)ds(y) \tag{6}$$

with a to-be-determined continuous density function φ, where

$$\Phi(x, y) := \frac{i}{4} H_0^{(1)}\big(k|x - y|\big), \quad x \neq y$$