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A simple distribution function-based gas-kinetic scheme for simulation of viscous incompressible and compressible flows

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ABSTRACT

In this work, a simple distribution function-based gas-kinetic scheme for simulation of viscous flows is presented. The work applies the finite volume method to discretize the governing differential equations, and inviscid and viscous fluxes at the cell interface are evaluated simultaneously by local reconstruction of solution for the continuous Boltzmann equation. Differently from the conventional gas-kinetic scheme [13–15], in the present work, the Maxwellian distribution function is simplified by a simple distribution function, and integrals in the infinity domain of phase space are reduced to integrals around a circle. As a consequence, the computational efficiency is greatly improved. Since the simple distribution function is defined on the circle, for simplicity, it is termed as circular function hereafter. The present work is the extension of our previous work [20], where the circular function-based gas-kinetic scheme is presented to simulate inviscid flows. Only the equilibrium distribution function is considered in [20]. To solve viscous flows, the non-equilibrium part of density distribution function has to be considered. One of major contributions in this work is to present a simple way to compute the non-equilibrium part of the distribution function. It can be calculated by the difference of equilibrium distribution functions at the cell interface and its surrounding point. As a result, the formulations for computing the conservative flow variables and fluxes at the cell interface can be given explicitly. The present solver can simulate both incompressible and compressible viscous flows. To validate the proposed new gas-kinetic scheme, several incompressible and compressible viscous flows are simulated. Numerical results showed that the circular function-based gas-kinetic scheme can provide accurate numerical results with the same computational cost as that needed by conventional Navier–Stokes solver.

1. Introduction

In recent years, the development of gas-kinetic scheme has attracted much attention. Differently from the Riemann solvers such as Roe scheme [1], van Leer scheme [2] and AUSM scheme [3], the gas-kinetic scheme constructs the numerical fluxes of macroscopic governing equations based on the local solution of Boltzmann equation. Since the continuum

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assumption is avoided in the Boltzmann equation, the solution of gas-kinetic scheme is more credible. That is why the gas-kinetic scheme is able to simulate both the continuous flow and rarefied flow. Another advantage of the gas-kinetic scheme is that the normal and tangential fluxes are treated consistently and the inviscid and viscous fluxes are considered as a whole. Furthermore, the gas-kinetic scheme can be used to simulate both incompressible and compressible flows without any modification in principle. Due to these advantages, the gas-kinetic scheme has received much attention to solve many engineering problems [4–7].

There are two common types of gas-kinetic schemes: Kinetic Flux Vector Splitting (KFVS) scheme and gas-kinetic Bhatnagar–Gross–Krook (BGK) scheme. Some of the representative KFVS schemes include the work of Pullin [8], Mandal and Deshpande [9] and Chou and Baganoff [10]. Basically, the KFVS scheme consists of two steps: free transport and collision. In the free transport step, the collisionless Boltzmann equation is solved to evaluate the fluxes. In the collision step, the collision process is actually controlled by a numerical time step. It has been demonstrated that the KFVS scheme provides good positivity property for simulation of flows with strong shock wave and expansion wave. Although it has some unique features, the KFVS scheme usually gives poorer results than those obtained from Godunov or Flux Difference Splitting (FDS) schemes [1,11]. As reported in the work of Xu [12], the main reason may be that the numerical dissipation of KFVS scheme is proportional to the mesh spacing. Consequently, only for those cases where the physical viscosity is larger than the numerical viscosity, the KFVS scheme could give accurate Navier–Stokes solutions.

With inclusion of BGK collision model, the gas-kinetic BGK scheme was proposed and developed by Prendergast and Xu [13], Chae et al. [14], Xu [15] and other researchers. In the BGK scheme, the particle transport is controlled by a real particle collision time, which is the function of the dynamic viscosity and pressure. In other words, instead of using the numerical time step to measure the particle collision process in the KFVS scheme, the real physical collision is considered in the BGK scheme. In the work of Xu [15], it has been demonstrated that the entropy condition is always satisfied by the BGK scheme since the gas evolution process is a relaxation process from a non-equilibrium state to an equilibrium one. In the smooth regions, the BGK scheme gives an accurate Navier–Stokes solution. In the discontinuous regions, a delicate dissipative mechanism in the BGK scheme generates a stable and crisp shock transition. On the other hand, as far as we know, most of the existing gas-kinetic schemes are based on the Maxwellian distribution function [5–7,13–18]. Due to complexity of the Maxwellian function, these gas-kinetic schemes are usually more complicated and less efficient than the traditional numerical schemes [1–3]. As mentioned in the work of Tang [19], for the viscous flow simulated by the BGK scheme, it is required to calculate a number of coefficients related to the physical space and phase space at every cell interface and each time step. This makes the evaluation of numerical fluxes by the BGK scheme be expensive. Moreover, as pointed out in [14], for the simulation of two-dimensional viscous flows, the BGK scheme requires about 1.5 times of the computational time needed by the conventional Navier–Stokes solver.

Recently, to improve the computational efficiency of the gas-kinetic scheme, a circular function-based gas-kinetic scheme was presented by Yang et al. [20]. Firstly, the original Maxwellian distribution function, which is a function of the phase velocity and phase energy, is simplified into the function of phase velocity only. The effect of the phase energy is embodied as particle potential energy. Then, the simplified Maxwellian function is further reduced to the circular function, which is a kind of the spherical harmonic functions, with assumption that all the particles are concentrated on a circle. By applying the circular function, a circular function based-KFVS scheme (CKFVS) and a circular function based-BGK scheme (CBGK) were presented for simulation of compressible inviscid flows [20]. Since no error and exponential functions, which are often appeared in the Maxwellian function-based gas-kinetic schemes [5–7,13–18], are involved in the circular function-based gas-kinetic schemes, the CKFVS scheme and CBGK scheme require less computational time than the corresponding Maxwellian function-based gas-kinetic schemes. On the other hand, it should be indicated that in our previous work [20], only the equilibrium distribution function is considered since we just target the simulation of inviscid flows. To solve viscous flows, the non-equilibrium part of density distribution function has to be incorporated. In this work, the non-equilibrium part of the distribution function is calculated by a simple way. That is, the non-equilibrium part is computed by the difference of equilibrium distribution functions at the cell interface and its surrounding point. Since the circular function is distributed along a circle rather than in an infinite domain, the equilibrium distribution functions at the surrounding points of the cell interface are actually distributed along the circle, which can be evaluated easily by interpolation. In addition, since no error and exponential functions are used, the present scheme is more efficient than the conventional gas-kinetic scheme [13–15]. Like other gas-kinetic schemes, the present scheme can be used to simulate both incompressible and compressible viscous flows without any modification. To validate the developed scheme, some viscous incompressible and compressible flows are solved. Numerical results showed that the present scheme can provide accurate results with roughly the same computational cost as required by the conventional Navier–Stokes solver [21].

2. Boltzmann equation, circular function and Navier–Stokes equations

2.1. Boltzmann equation and circular function

In the application of gas-kinetic scheme, the Bhatnagar–Gross–Krook (BGK) collision model [22] is widely used. The continuum Boltzmann equation with BGK collision model can be written as

$$\frac{\partial f}{\partial t} + \xi \cdot \nabla f = \frac{g - f}{\tau} \tag{1}$$