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# A refined volume-of-fluid algorithm for capturing sharp fluid interfaces on arbitrary meshes

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ABSTRACT

This paper presents a new volume-of-fluid scheme (M-CICSAM), capable of capturing abrupt interfaces on meshes of arbitrary topology, which is a modification to the compressive interface capturing scheme for arbitrary meshes (CICSAM) proposed in the recent literature. Without resort to any explicit interface reconstruction, M-CICSAM is able to precisely model the complex free surface deformation, such as interface rupture and coalescence. By theoretical analysis, it is shown that the modified CICSAM overcomes three inherent drawbacks of the original CICSAM, concerning the basic differencing schemes, the switching strategy between the compressive downwind and diffusive high-resolution schemes, and the far-upwind reconstruction technique on arbitrary unstructured meshes. To evaluate the performance of the newly proposed scheme, several classic interface capturing methods developed in the past decades are compared with M-CICSAM in four test problems. The numerical results clearly demonstrate that M-CICSAM produces more accurate predictions on arbitrary meshes, especially at high Courant numbers, by reducing the numerical diffusion and preserving the interface shape.



## 1. Introduction

Numerical simulation of immiscible multi-fluid flows has been an active area of research over the past decades, due to the common presence of such flows in nature and their importance in many practical applications [1–3]. An effective numerical scheme should be able to handle a number of distinct flow features properly, such as high density ratios, large pressure differences at fluid–fluid interfaces, and the evolution of the free surface. Various numerical approaches for computing immiscible multi-fluid flows have been proposed in the literature; they can be grouped into three broad categories: interface/surface fitting methods, interface/surface tracking methods and interface/surface capturing methods [4–6].

Interface/surface fitting methods [7–11] are associated with boundary-fitted moving grids, where the grid points are attached to the fluid particles and move with them in a Lagrangian manner. Generally, these methods solve for the flow only within a single fluid region and treat the free surface as a free-floating boundary. The advantage of these methods is that they allow a precise representation of the interfacial jump conditions and thus maintain a sharp interface, whose exact position is known throughout the calculation. However, large interface deformations will result in highly distorted meshes. Therefore, special procedures need to be designed in order to prevent both grid singularities and extremely skewed grid

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point distributions, which impose restrictions on their applications. Both interface tracking and interface capturing methods adopt fixed Eulerian meshes and are particularly well-suited for dealing with large interface deformations [12,13].

In interface/surface tracking methods, such as mark and cell (MAC) schemes and geometric-type VOF schemes, the interface is explicitly approximated either by massless marker particles or by an indicator function. In the MAC methods, the free-surface location is determined by introducing massless marker particles that move with the instantaneous velocity field. In general, such methods are non-conservative and computationally expensive [14–16]. In the geometric-type VOF methods, the interface is explicitly reconstructed by volume fraction values, which is also used in designing the scalar advection scheme. Different researchers have proposed various algorithms of this kind, such as the simple line interface calculation (SLIC) method and the piecewise linear interface calculation (PLIC) method [17–21]. These methods are capable of maintaining a very sharp interface while at the same time ensuring mass conservation. However, the processes of reconstructing and tracking the interface at each time step remain complicated and challenging, especially when extending to unstructured meshes and to three dimensions.

Interface/surface capturing methods, such as artificial compressibility schemes, level set schemes and algebraic-type VOF schemes, are characterized by algebraically solving the scalar transport equation without explicitly reconstructing the interface, which overcome the main drawbacks of interface tracking methods. In artificial compressibility schemes [4,6,22], the free surface location is automatically captured as a contact discontinuity in the density field by enforcing conservation laws, thus eliminating the need for reconstructing and tracking the interface. In level set schemes [23,24], the interface is defined as the zero-contour of a distance function, which is advected with the local flow field. Although being conceptually simple and relatively easy to implement, level set techniques suffer from the errors in the mass conservation principle.

Algebraic-type VOF methods include high-resolution differencing schemes, flux-limited methods, inter-gamma schemes, analytical-function fitted methods and blended high-resolution differencing methods. High-resolution differencing schemes [2,25–27] are inclined to introduce excessive numerical diffusion and dispersion. In inter-gamma schemes [28,29], an extra artificial compressive term is added into the VOF advection equation for the purpose of compressing the interface, instead of just employing a compressive differencing scheme. Flux-limited methods [30–32] consist of a basic high-resolution advection scheme and a multi-dimensional flux limiter, which are conservative, monotonic and shape-preserving for both continuous and discontinuous density fields. In analytical-function fitted methods [33,34], different smooth basic functions, such as the hyperbolic tangent function and the cubic polynomial function, are adopted to represent a discontinuity at the grid scale in the flux computation of the volume fraction.

A compressive downwind scheme can maintain the sharpness of the interface, but it tends to distort the interface when the flow is not aligned with the computational mesh. On the contrary, a more diffusive high-resolution scheme often turns out to be too diffusive. In order to remedy the aforementioned two problems, blended high-resolution differencing methods make use of the NVD/NVSF concept and switch continuously between a compressive downwind and a diffusive high-resolution scheme, according to the angle between the interface direction and the grid orientation. Various blended high-resolution methods have been proposed in the literature, including CICSAM [1], HiRAC [3], STACS [5], SURFER [35], HRIC [36], FBICS [37] and THOR [38]. These methods are computationally efficient, strictly conserve mass, and can be easily extended to unstructured meshes and three dimensions, so they are capable of accurately capturing the free surface and modelling merging and fragmentation in multiphase flows.

In this article, a new blended high-resolution differencing method (M-CICSAM) is presented, whose accuracy is compared with four existing schemes of the same kind. The results obtained in all the test cases clearly show the advantage of M-CICSAM.

## 2. The governing equations

The computational grids across the interface are occupied by different immiscible fluids, which are treated as a single effective fluid with continuous physical properties. The density $\rho$ and dynamic viscosity $\mu$, across the interface are evaluated using the following relations:

$$\rho = \sum_{i=1}^{n} \alpha_i \rho_i; \qquad \mu = \sum_{i=1}^{n} \alpha_i \mu_i; \qquad \sum_{i=1}^{n} \alpha_i = 1 \tag{1}$$

where the subscript $i$ denotes the $i$th fluid, and $\alpha$ is the volume fraction defined as the volume percentage of the $i$th fluid available in a cell. The effective fluid is presumed to obey the same set of governing equations as a single fluid:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \tag{2}$$

$$\frac{\partial \rho u_i}{\partial t} + \nabla \cdot (\rho u_i u_j) = \nabla \cdot T + \rho g_i + f_{\sigma i} \tag{3}$$

where the subscripts $i$ and $j$ indicate the $i$th and $j$th directions of the Cartesian coordinate system respectively, $u$ is velocity, $g$ is gravitational acceleration, $t$ is time, $f_\sigma$ is the surface tension, and $T$ is the stress tensor which contains the pressure $P$.