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Introducing a new kinetic model which admits an H-theorem for simulating the nearly incompressible fluid flows

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ABSTRACT

We present a new kinetic model, for simulating the nearly incompressible fluid flows. The model uses constant speed particles which are free to move in all directions. In the new model, the Maxwellian is replaced by a fractional form, while the Navier–Stokes and continuity equations are recovered. It is known that the conventional lattice Boltzmann model (LBM) with polynomial equilibrium distribution function cannot admit an H-theorem (Wagner, 1998) [9]. In the present work, we show that the new model admits an H-theorem and the numerical schemes which stem from the new model are more stable than the conventional LBM. For the streaming stage, two different approaches, namely the lattice based and the discontinuous Galerkin based schemes, are introduced. The former is more cost effective than the conventional lattice Boltzmann models while it maintains their simplicity. The latter is a higher order spectral element method, using which one can employ non-uniform triangular elements with high accuracy and geometrical flexibility. Unlike the conventional LBM, the new model is derived from continuous relations. Hence, it does not require a symmetric discrete velocity model. The accuracy and stability of the model have been verified, by simulating three benchmark problems. The plane Couette flow, lid driven square cavity, and flow around an impulsively started cylinder have been simulated. The results of the present work are in excellent agreement with the exact solutions and with the results reported by others.

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1. Introduction

The statistical behavior of a dilute gas can be studied by employing the Boltzmann equation [1]. The Chapman–Enskog analysis [2] shows that the first and second order approximations of the Boltzmann equation result in the Euler and Navier–Stokes equations, respectively. For the meso and macro scale flows, the microscopic details are not needed and the flows can adequately be described by the Navier–Stokes equations; therefore, the rare gas assumption is not necessary and one may use the Boltzmann equation to study such flows. The kinetics based lattice Boltzmann models have been considered as alternatives to the continuum based Navier–Stokes equations, during the past two decades. The most important advantages of the kinetics based models over the traditional continuum based models are as follows: (1) relative simplicity, (2) ease of handling complex geometries, and (3) the local nature of the algorithms which makes the kinetics based models an excellent choice for fast parallel computing.

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The conventional LBM with polynomial equilibrium distribution function, however, still faces some challenges in both compressible and incompressible flow regimes. For example, lack of H-theorem is known to be, at least in part, responsible for instabilities at relatively low Mach numbers [3]. The instability which occurs in the simulation of single phase multi-component (SPMC) flows with high density ratio is another important example of open issues in the LBM [4,5]. The main interest of the present work is to improve the stability of the conventional LBM.

Because of the complicated structure of the collision integral of the Boltzmann equation, it is customary to replace the collision term with simpler alternative expressions which are known as the collision models. Any Boltzmann-like equation, where the Boltzmann collision integral is replaced by a collision model, is called a model equation or a kinetic model [1]. The most widely known collision model is the Bhatnagar–Gross–Krook (BGK) model, which was independently proposed by Welander at about the same time [1]. To further simplify the kinetic models, the discrete velocity models [6] were proposed, in which a finite set of velocities represent the entire velocity space. In all kinetic models, two key issues are the determination of the collision and streaming stages.

In the collision stage, determination of the equilibrium distribution function (EDF) is the most important task. The Maxwellian equilibrium distribution is compatible with an H-theorem, yet it cannot be used in the discrete velocity models [7]. For the lattice Boltzmann models with polynomial equilibrium distribution function, on the other hand, no H-theorem exists, and the equilibrium is an attractor, rather than a true equilibrium [7]. Hence, the stability of the models can only be studied by means other than proving an H-function [7]. Sterling et al. [8] applied Von Neuman's stability analysis method to the linearized collision operator of some lattice Boltzmann models. Wagner [9] showed that the usual choice of polynomial equilibrium distribution function is incompatible with an H-theorem, for the lattice Boltzmann models. Yong and Li-Shi Luo [10] proved the non-existence of the H-theorem, for some lattice Boltzmann models. They derived a consistency equation that must be satisfied by a lattice Boltzmann scheme for which an H-theorem exists. The entropic lattice Boltzmann models (ELBM) have been introduced, to improve the stability of the standard lattice Boltzmann models, by restoring the H-theorem [3,11–13].

For the streaming stage, various methods have been proposed. In the LBM, the velocity and physical spaces are coupled and the outcomes of the collisions are simply transferred to the neighboring cells. While this approach greatly reduces the computational costs, it limits the model to unit CFL (Courant–Friedrich–Lewy) and uniform grids. It was shown that the lattice Boltzmann equation (LBE) is a special discretization of the continuous Boltzmann equation [14,15]. Therefore, the velocity and physical spaces can be decoupled. Since then, different numerical methods, such as the finite difference (FD) [16], finite volume (FV) [17], finite element (FE) [18], and spectral element methods (SEM) [19], have been applied to the LBM. A discontinuous Galerkin spectral element method (DGSEM), based on triangular elements, was implemented to solve the LBE [20], where modified Dubiner triangular basis [21] was used as the test function. This model was employed to simulate flow past a circular cylinder at relatively low Reynolds numbers. By decoupling the collision step from the streaming step, Min et al. [22] proposed a discontinuous Galerkin spectral element method to solve the lattice Boltzmann equation, and they showed that their numerical scheme can be used to simulate flows at high Reynolds numbers, with improved numerical stability.

He and Li-Shi Luo [14] derived the lattice Boltzmann models directly from the continuous Boltzmann equation. Their work included the $d2q6$ model (without rest particles) and $d2q7$ model (with rest particles), where constant speed particles collide and stream on a hexagonal lattice structure. Zheng et al. [23] proposed a platform, using which new equilibrium distribution functions for some constant speed discrete velocity models can be constructed. Qu et al. [24,25] introduced a new and simple form for the equilibrium distribution function, which they referred to as the "circular function". The particles, in their model, are assumed to be concentrated on a circle centered at (u_x, u_y) where u_x and u_y are the components of the macroscopic velocity vector at the target node. The collision circle, therefore, changes position from time to time and according to the macroscopic velocity vector. In a recent paper, Yang et al. [26] showed that the circular function can be derived from the Maxwellian function. The circular model is justified because it recovers the Navier–Stokes equations. The Lagrange multiplier's technique is used to project the continuous equilibrium distribution function into a finite and discrete set of polynomial functions of the local state variables. Wang et al. [27] proposed another type of "circular function", which they referred to as the "polynomial kernel function". The equilibrium distribution corresponding to various discrete velocities, in their model, are also polynomial functions of the state variables. Their model can be used to simulate viscous compressible flows with flexible specific heat ratio and Prandtl numbers. The non-existence of an H-theorem for the lattice Boltzmann models with polynomial equilibrium distribution function has been proved by Wagner [9]. Thus, none of the above mentioned constant speed models admit an H-theorem.

In this work, we present a new kinetic model, for simulating the nearly incompressible fluid flows. The model uses constant speed particles which are free to move in all directions. In Section 2, the Maxwell–Boltzmann equilibrium distribution is replaced by a fractional form, the incompressible Navier–Stokes equation is recovered, and expressions for the pressure and viscosity are found. Moreover, we show that the present model admits an H-theorem. By inserting the rest particles into the collision rule, the pressure and the speed of sound of the model can be regulated. For the streaming stage, two alternatives, namely the lattice based and the discontinuous Galerkin (DG) based approaches, are introduced, analyzed, and tested. In Section 3, the accuracy and the stability of the model have been verified, by simulating three well known test cases. Plane Couette flow, lid driven square cavity at Reynolds numbers ranging from 400 to 7500, and flow around an impulsively started cylinder at Reynolds numbers of 550 and 9500 are simulated. The results of the present simulations are in excellent agreement with the results reported by others and with the analytical and exact solutions. In Section 4,

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