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Combining perturbation theory and transformation electromagnetics for finite element solution of Helmholtz-type scattering problems

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ABSTRACT

A numerical method is proposed for efficient solution of scattering from objects with weakly perturbed surfaces by combining the perturbation theory, transformation electromagnetics and the finite element method. A transformation medium layer is designed over the smooth surface, and the material parameters of the medium are determined by means of a coordinate transformation that maps the smooth surface to the perturbed surface. The perturbed fields within the domain are computed by employing the material parameters and the fields of the smooth surface as source terms in the Helmholtz equation. The main advantage of the proposed approach is that if repeated solutions are needed (such as in Monte Carlo technique or in optimization problems requiring multiple solutions for a set of perturbed surfaces), computational resources considerably decrease because a single mesh is used and the global matrix is formed only once. Only the right hand side vector is changed with respect to the perturbed material parameters corresponding to each of the perturbed surfaces. The technique is validated via finite element simulations.

1. Introduction

Ever since the invention of computers, several analytical and numerical techniques have enjoyed wide success in solving the problem of electromagnetic scattering by obstacles. Among those techniques, finite methods (finite element method or finite difference methods) have been preferred because of their adaptability to arbitrary geometries and material inhomogeneities. However, these methods are not ideally suited to certain problems requiring repeated solutions due to large computational resources. For example, if the analysis of shape perturbation, such as surface roughness, is to be performed by a stochastic or statistical approach (e.g., Monte Carlo approach), or if a shape optimization problem is to be solved, repeated solutions are required to get a family of data for different rough surfaces or shapes. Specifically, after producing a set of surface geometries, a mesh must be generated with respect to each of these geometries, and a matrix system must be formed and solved anew for each mesh. This will obviously increase the computation time and data storage given the large number of realizations.

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The main purpose of this study is to propose an approach combining the perturbation theory and the transformation electromagnetics to easily handle repeated computations by employing a *single* mesh and forming the matrix *only once* in the finite element solution of electromagnetic scattering from obstacles having shape perturbations. Perturbation theory is basically a class of techniques to find the effect of "small" changes on known solutions by adding a "small" term to the mathematical description of the problem. Depending on each specific application or discipline, it has different forms; but it is especially useful for understanding weak imperfections or interactions. Numerical characterization of weak perturbations on the surface of an obstacle has long been of interest to researchers not only in electromagnetics but also in acoustics, mechanics, etc. Particularly, the perturbation theory has been used with different interpretations for rough surface scattering [1–9] and for shape optimization [10–12] problems. The proposed approach in this paper presents an alternative interpretation of the perturbation theory by incorporating it with the salutary features of the transformation electromagnetics/optics. The introduction of the transformation electromagnetics/optics approach (formerly known as coordinate transformation technique that employs the form invariance property of Maxwell's equations) can be traced back to the mathematician Bateman at the beginning of the 20th century [13]. The central concept of this approach has been discussed in some books [14,15] and in the context of finite methods [16,17]. This approach has also been used in the design of perfectly matched layers (PMLs) for the purpose of mesh truncation in finite methods [18,19] (this is known as coordinate stretching in PML nomenclature). However, this intuitive approach has evolved as a widely-used approach for systematic design of various optical and electromagnetic structures [20–33] especially after the introduction of the invisibility cloak [20]. The main idea of the transformation electromagnetics is that if the spatial space of a medium is distorted by a coordinate transformation, this medium is equivalent to an anisotropic medium in which Maxwell's equations keep the same mathematical form (this property is known as the form invariance of Maxwell's equations under coordinate transformations). The material tensors representing the constitutive parameters of the anisotropic medium are determined by the Jacobian of the coordinate transformation.

In the light of above general descriptions of the transformation electromagnetics and the perturbation theory, how these two approaches are hybridized to model electromagnetic scattering from obstacles with perturbed surfaces can be outlined as follows: (i) First, an obstacle with perfectly smooth surface is considered. In the finite element method (FEM) solution of this problem, a mesh is created and the matrix system is solved for the unknown fields. (ii) Using the same mesh for the smooth surface, the obstacle is coated by a transformation medium layer. To obtain the constitutive parameters (material tensor) of this medium, a special coordinate transformation is defined, which maps the smooth surface to the perturbed surface. Note that this mapping is only "virtual", meaning that the coordinates of the mesh are not altered and the effect of this transformation is preserved in the material parameters. In other words, there is a correspondence between the perturbed surface of the object and the resulting material tensor ($\bar{\bar{\Lambda}}$) after the coordinate transformation. In the absence of perturbation (i.e., in the case of smooth surface), the material tensor becomes an identity tensor. (iii) Assuming that the perturbation on the surface is small, the perturbation in the material tensor (deviation from the identity tensor, $\delta\bar{\bar{\Lambda}}$) and the perturbation in the fields (deviation from the fields of the smooth surface, $\delta\mathbf{E}$, $\delta\mathbf{H}$) will also become small. When the fields and the material tensor are substituted into Maxwell's equations, it has been found that the perturbed fields also satisfy Maxwell's equations in the original medium of the problem, and the perturbed tensor together with the fields of the smooth surface appear as "source" terms. These source terms will come out on the right hand side (RHS) of the FEM matrix system. Therefore, without re-meshing and re-forming the global matrix, only the RHS vector is modified with respect to the perturbed material tensor pertaining to the perturbed surface. The most significant feature of this result is that multiple RHS vectors (i.e., multiple perturbed surfaces) can be handled efficiently with little computational burden. For example, if the global matrix is factorized by LU decomposition, then the repeated solutions can be generated by using only forward and backward substitutions, without resorting to Gaussian elimination each time. This feature considerably decreases the computational resources especially in the Monte Carlo technique or in optimization problems where multiple solutions are needed.

The paper is organized as follows: Section 2 presents the formulation of the proposed approach in conjunction with the FEM solution of the Helmholtz equation. Note that the analysis is given in 2D, but can be generalized to 3D problems in a straightforward manner because the derivation of the material parameters is valid in 3D space as well. In Section 3, the results of some numerical simulations are reported. Finally, Section 4 draws some conclusions.

2. Formulation

Consider a typical electromagnetic scattering problem involving a conducting object with *smooth* surface and illuminated by a plane wave with arbitrary direction of incidence, as shown in Fig. 1(a). The open-region of the computational domain is terminated by a perfectly matched layer (PML), which is implemented by locally-conformal PML approach [19]. Assuming that the suppressed time-dependence is $\exp(j\omega t)$, the source-free Maxwell's equations are expressed as follows:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \tag{1a}$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} \tag{1b}$$

where ε and μ are, respectively, the permittivity and permeability of the medium (typically free-space). By manipulating Maxwell's equations, the vector wave equation for the electric field can be written as follows: