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A preconditioned dual–primal finite element tearing and interconnecting method for solving three-dimensional time-harmonic Maxwell's equations

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ABSTRACT

A new preconditioned dual–primal nonoverlapping domain decomposition method is proposed for the finite element solution of three-dimensional large-scale electromagnetic problems. With the aid of two Lagrange multipliers, the new method converts the original volumetric problem to a surface problem by using a higher-order transmission condition at the subdomain interfaces to significantly improve the convergence of the iterative solution of the global interface equation. Similar to the previous version, a global coarse problem related to the degrees of freedom at the subdomain corner edges is formulated to propagate the residual error to the whole computational domain at each iteration, which further increases the rate of convergence. In addition, a fully algebraic preconditioner based on matrix splitting is constructed to make the proposed domain decomposition method even more robust and scalable. Perfectly matched layers (PMLs) are considered for the boundary truncation when solving open-region problems. The influence of the PML truncation on the convergence performance is investigated by examining the convergence of the transmission condition for an interface inside the PML. Numerical examples including wave propagation and antenna radiation problems truncated with PMLs are presented to demonstrate the validity and the capability of this method.

1. Introduction

For computational electromagnetics, the finite element method (FEM) is a powerful numerical tool for its excellent capability to model complex media and delicate geometries [1,2]. However, for the analysis of large-scale three-dimensional (3D) problems, finite element simulations usually require massive computational resources and result in a linear system difficult to solve efficiently no matter using a direct or an iterative solver. To overcome this challenge, several finite-element-based domain decomposition methods (DDMs) have been proposed to artificially split the entire computational domain into smaller subdomains and hybridize direct and iterative solvers in a two-level manner during the solution procedure [3–6]. These algorithms are inherently suitable for parallel implementation, thus taking the full advantage of new generation parallel processors and clusters. The two most advanced nonoverlapping DDMs are the dual–primal finite element tearing and interconnecting (FETI-DP) method [4,5,7,8] and the cement element method [6,9,10], which stem from the Schur complement method [3] and the Schwarz method [3], respectively. Both methods were initially developed and have been widely employed for solving scalar Helmholtz equations [11–16].

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The interface transmission condition (TC) used to connect the subdomains is recognized as a key ingredient affecting the convergence performance of a nonoverlapping domain decomposition algorithm when solving a wave equation. This interface condition should make the subdomain interfaces as transparent as possible to facilitate information exchange across the subdomains for a fast iterative convergence of the global interface problem. To improve the widely used first-order TC (FOTC) [5,6], a few higher-order TCs have been derived by using the Fourier analysis [15–18]. However, some of these interface conditions are nonlocal in nature and have to be approximated by local operators. Among those designed for vector wave equations, the optimized TC based on first-order Maxwell's equations [19], the approximated sparse boundary integral (BI) equation [20], the second-order transverse-electric TC (SOTC-TE) [9,17], and the fully second-order TC (SOTC-FULL) [10] have demonstrated an improved performance. Particularly, the SOTC-FULL ensures the transmission of both transverse-electric (TE) and transverse-magnetic (TM) evanescent modes, without sacrificing the transmission of propagating modes, by adding to the FOTC a surface curl–curl term related to the interface electric field and another surface gradient corresponding to the interface electric charge density [10]. On the other hand, at the discrete level, it has been observed that the optimal interface operator is equal to the Schur complement of the outer domain [21,22], which can be directly approximated using purely algebraic techniques like sparse approximate inverse methods or incomplete factorization [21]. However, this approach is limited to the case with conformal meshes on the subdomain interfaces. Recently, a new technique called the sweeping preconditioner was advocated for solving time-harmonic wave equations [23], where several perfectly matched layers (PMLs) were employed to mimic the behavior of transparent interface conditions. Due to its overlapping nature, this algorithm does not scale well with respect to the number of subdomains.

Another issue associated with a DDM is the treatment of corner unknowns which are defined on the geometrical crosspoints shared by more than two subdomains. When such crosspoints are present, one must weakly impose two different sets of equations for each interface at the crosspoint, thus making the discretization more complicated [24–27]. It has been found that the TC parameters need to be modified for the interface containing geometrical crosspoints when solving the scalar Helmholtz equations [25]. A similar difficulty was also observed in the case of Maxwell's equations [10], when the surface curl-conforming vector basis functions were adopted to expand the cement variables. One has to break cement variables defined on corner edges into two independent components to avoid an incorrect enforcement of the tangential magnetic field continuity at the corner. Additional corner edge penalty terms, relating to the divergence-free constraint for the cement variables, have to be introduced to remove singularity caused by redundant cement variables. In contrast, the FETI-DP method solves this problem efficiently by extracting corner electric fields out and constructing a global corner system [4,5,7] by enforcing a strong Dirichlet continuity condition rather than a weak TC at corners. The purpose of such a coarse grid correction is two-fold: (1) It avoids redundant auxiliary variables at corner edges because no dual unknowns have to be defined there, and (2) it introduces a mechanism to propagate the iterative residual error globally. The FOTC has been incorporated into this dual–primal strategy [5,7] and demonstrated a remarkable performance. However, a higher-order TC has yet to be incorporated into this dual–primal scheme.

As the global interface problem of a DDM is solved iteratively, a preconditioner is always desirable to improve the iterative convergence. However, application of the well-investigated algebraic preconditioners, such as those based on a partial inverse, is difficult because the matrix associated with the global interface problem is not explicitly assembled. To efficiently solve scalar Helmholtz problems, one can devise a preconditioner by constraining the residual of the iterative FETI solution to be orthogonal to an auxiliary wave-based coarse space [28,29]. Enlightened by this idea, a similar preconditioning technique, called global plane wave deflation (GPWD), was derived to alleviate the weakly convergent influences caused by cutoff or near-cutoff modes [30]. Different from the one proposed in [28], the GPWD preconditioner constructs an auxiliary coarse space on subdomain interfaces rather than within the entire computational domain. As a result, geometrically planar subdomain interfaces are required to support surface plane waves. More recently, a fully algebraic local preconditioner called the locally exact algebraic preconditioner (LEAP) was proposed to accelerate the FETI method [31]. A careful study shows that with the global interface unknowns reordered, the LEAP is essentially a block diagonal preconditioner involving computationally expensive preprocessing. In this paper, we present an efficient preconditioning scheme based on matrix splitting for the FETI-DP method. This preconditioner does not require assembling the global interface system matrix explicitly, thus having a high parallel efficiency. In addition, it consumes only a negligible extra memory and computation time in the setup stage.

The rest of this paper is organized as follows. In Section 2.1, we first discuss the choice of TC parameters for the case of PML truncation. Because the PML material could be active along a certain axis though it is lossy in general, the interface TC has to be designed carefully to avoid any mismatch or divergence if the subdomain interface resides inside the PML. Then, we formulate a new FETI-DP method combining the dual–primal idea with higher-order TCs to significantly improve the convergence of the interface solution in Section 2.2. Afterwards, a matrix splitting technique is introduced in Section 2.3 to facilitate the construction of an efficient preconditioner to make the new FETI-DP method more robust. Finally, several wave propagation and antenna radiation examples are shown to illustrate the better convergence performance of the proposed method by comparing it with several other DDM solvers in Section 3.