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A multi-domain Fourier pseudospectral time-domain method for the linearized Euler equations

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ABSTRACT

The Fourier pseudospectral time-domain (F-PSTD) method is computationally one of the most cost-efficient methods for solving the linearized Euler equations for wave propagation through a medium with smoothly varying spatial inhomogeneities in the presence of rigid boundaries. As the method utilizes an equidistant discretization, local fine scale effects of geometry or medium inhomogeneities require a refinement of the whole grid which significantly reduces the computational efficiency. For this reason, a multi-domain F-PSTD methodology is presented with a coarse grid covering the complete domain and fine grids acting as a subgrid resolution of the coarse grid near local fine scale effects. Data transfer between coarse and fine grids takes place utilizing spectral interpolation with super-Gaussian window functions to impose spatial periodicity. Local time stepping is employed without intermediate interpolation. The errors introduced by the window functions and the multi-domain implementation are quantified and compared to errors related to the initial conditions and from the time iteration scheme. It is concluded that the multi-domain methodology does not introduce significant errors compared to the single-domain method. Examples of scattering from small scale density scatters, sound reflecting from a slitted rigid object and sound propagation through a jet are accurately modelled by the proposed methodology. For problems that can be solved by F-PSTD, the presented methodology can lead to a significant gain in computational efficiency.

1. Introduction

To obtain deterministic time-dependent solutions for physical problems as unsteady fluid flow and acoustic propagation through a non-uniform medium, numerical solution strategies using a volumetric discretization of the spatial domain are needed. These solution strategies often share geometrical and time-domain limitations due to the required computational resources. The development of numerical methodologies that keep accuracy but enhance computational efficiency is challenging many branches of the computational engineering society. One of the techniques to solve time-dependent problems, that has received significant interest over the last decades, is the family of pseudospectral time-domain (PSTD) methods, see e.g. Refs. [1–5]. The most efficient of the PSTDs, the Fourier PSTD method (F-PSTD) has similarities with the well-known finite-difference time-domain (FDTD) method [6], as the physical domain of interest is discretized by an orthogonal equidistant grid and the solution is sought at discrete grid points. For the evaluation of the time-derivatives in FDTD and F-PSTD, numerical methods as the Runge–Kutta method, Adams–Bashforth method or MacCormack scheme can be used. In contrast to using finite differences, evaluation of the spatial derivatives in F-PSTD is carried out by transforming the spatial variables

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through a set of basis functions and executing the derivatives in the transformed domain. An inverse transformation through the basis functions returns the spatial derivatives in the spatial domain. As the basis in F-PSTD consists of Fourier kernels, the transforms can be operated by fast Fourier transforms (FFT). As a result, only 2 spatial points per smallest wavelength are necessary to resolve the smallest wavelength of interest with spectral accuracy. This low resolution can even be approximated for slowly varying medium properties [7]. The F-PSTD method herewith offers, compared to the FDTD method with similar accuracy, a way to reduce the number of degrees of freedom as well as the computational efficiency, especially for three-dimensional problems, see e.g. [1,8]. It is at the other hand limited by its uniform Cartesian grid and so far, impedance boundary conditions can only be treated in an approximate way, see e.g. [9]. This paper focuses on a solution of acoustic propagation problems governed by the linearized Euler equations (LEE) in non-conservative form. In three-dimensional (3D) Cartesian coordinates, they read:

$$
\begin{aligned}
&\frac{\partial \mathbf{q}}{\partial t} + \mathbf{A}_j \frac{\partial \mathbf{q}}{\partial j} + \mathbf{C}\mathbf{q} = \mathbf{0},\\
&\mathbf{A}_j = \begin{bmatrix} u_{0,j} & \rho_0\delta_{x,j} & \rho_0\delta_{y,j} & \rho_0\delta_{z,j} & 0 \\ 0 & u_{0,j} & 0 & 0 & \frac{\delta_{x,j}}{\rho_0} \\ 0 & 0 & u_{0,j} & 0 & \frac{\delta_{y,j}}{\rho_0} \\ 0 & 0 & 0 & u_{0,j} & \frac{\delta_{z,j}}{\rho_0} \\ 0 & \gamma p_0\delta_{x,j} & \gamma p_0\delta_{y,j} & \gamma p_0\delta_{z,j} & u_{0,j} \end{bmatrix},\\
&\mathbf{C} = \begin{bmatrix} \frac{\partial u_{0,j}}{\partial j} & \frac{\partial \rho_0}{\partial x} & \frac{\partial \rho_0}{\partial y} & \frac{\partial \rho_0}{\partial z} & 0 \\ \frac{u_{0,j}}{\rho_0}\frac{\partial u_{0,x}}{\partial j} & \frac{\partial u_{0,x}}{\partial x} + D & \frac{\partial u_{0,x}}{\partial y} & \frac{\partial u_{0,x}}{\partial z} & 0 \\ \frac{u_{0,j}}{\rho_0}\frac{\partial u_{0,y}}{\partial j} & \frac{\partial u_{0,y}}{\partial x} & \frac{\partial u_{0,y}}{\partial y} + D & \frac{\partial u_{0,y}}{\partial z} & 0 \\ \frac{u_{0,j}}{\rho_0}\frac{\partial u_{0,z}}{\partial j} & \frac{\partial u_{0,z}}{\partial x} & \frac{\partial u_{0,z}}{\partial y} & \frac{\partial u_{0,z}}{\partial z} + D & 0 \\ 0 & \frac{\partial p_0}{\partial x} & \frac{\partial p_0}{\partial y} & \frac{\partial p_0}{\partial z} & \gamma\frac{\partial u_{0,j}}{\partial j} \end{bmatrix},\\
&D = \frac{\partial u_{0,j}}{\partial j} + \frac{u_{0,j}}{\rho_0}\frac{\partial \rho_0}{\partial j},
\end{aligned} \tag{1}
$$

with $\mathbf{q} = [\rho, u_x, u_y, u_z, p]^T$ the acoustic variable vector, ρ the density, u_j the velocity components with index j equal to x, y or z, p the pressure, γ the heat capacity ratio and δ the Kronecker delta function. All physical variables are decomposed into their ambient values, denoted by subscript 0, and acoustic fluctuations:

$$
\begin{aligned}
\rho_{\text{tot}} &= \rho_0 + \rho,\\
\mathbf{u}_{\text{tot}} &= \mathbf{u}_0 + \mathbf{u},\\
p_{\text{tot}} &= p_0 + p,
\end{aligned} \tag{2}
$$

with $\mathbf{u} = [u_x, u_y, u_z]^T$ the velocity vector. Eqs. (1) assume that the ambient variables are known. No constraints on the compressibility of the background medium are imposed, and no aero-acoustic sources are considered. Eq. (1) are closed with boundary conditions and enable to solve acoustic propagation problems with arbitrary media properties, including the presence of a non-uniform mean flow. A large amount of these problems are characterized by smoothly varying ambient properties and acoustically rigid staircase-type boundaries[1] and can be solved by the extended F-PSTD method, e.g. outdoor sound propagation in the presence of an atmospheric wind field and detailed rigid objects as trees, and flow acoustic problems such as sound radiation from a pipe in the presence of a jet flow. The F-PSTD discretization of these problems is determined by the smallest wavelength of interest, the smallest length scales from geometry or medium heterogeneities. For many applications, the equidistant discretization implies a too high discretization to resolve the smallest wavelength of interest for most of the domain. Such applications would preferably be solved by a multi-domain discretization method. A multi-domain F-PSTD method would favour hybrid domain discretization methods involving other methods, as the F-PSTD method excels regarding both the required number of grid points per wavelength and stability conditions for time-dependent problems. Wang and Takenaka have presented such a methodology [10]. Their work contained a sequence of domains along one direction and a global time step bounded by the smallest spacing was used throughout the domain. For interpolation from the coarse to the fine grid, spectral interpolation was implemented globally, i.e. for the complete domain.

In this paper, a multi-domain F-PSTD method to solve the LEE is presented where the sub-domains differ in grid spacing and are separated by interfaces. Data transfer between the grids takes place at both sides of the interface. The coarse grid variable values are spectrally interpolated in the spatial domain to the finer grid, and the fine grid variable values are

[1] Staircase-type boundaries denote boundaries that can be captured by an equidistant orthogonal grid.