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Numerical study of the two-species Vlasov–Ampère system: Energy-conserving schemes and the current-driven ion-acoustic instability

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ABSTRACT

In this paper, we propose energy-conserving Eulerian solvers for the two-species Vlasov–Ampère (VA) system and apply the methods to simulate current-driven ion-acoustic instability. The two-species VA systems are of practical importance in applications, and they conserve many physical quantities including the particle number of each species and the total energy that is comprised of kinetic energy for both species and the electric energy. The main goal of this paper is to generalize our previous work for the single-species VA system [9] and Vlasov–Maxwell (VM) system [8] to the two-species case. The methodologies proposed involve careful design of temporal discretization and the use of the discontinuous Galerkin (DG) spatial discretizations. We show that the energy-conserving time discretizations for single-species equations [9,8] can also work for the two-species case if extended properly. Compared to other high order schemes, we emphasize that our schemes can preserve the total particle number and total energy on the fully discrete level regardless of mesh size, making them very attractive for long time simulations. We benchmark our algorithms on a test example to check the one-species limit, and the current-driven ion-acoustic instability. To simulate the current-driven ion-acoustic instability, a slight modification for the implicit method is necessary to fully decouple the split equations. This is achieved by a Gauss–Seidel type iteration technique. Numerical results verified the conservation and performance of our methods. Finally, we remark that the schemes in this paper can be readily extended to applications when the models take more general form, such as the multi-species VM equations.

1. Introduction

In this paper, we propose energy-conserving Eulerian solvers for the two-species Vlasov–Ampère (VA) system and apply the methods to simulate current-driven ion-acoustic instability. The two-species VA model describes the evolution of the distribution functions for a single species of electrons and ions under the influence of the self-consistent electric field.

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Accurate numerical simulation for this system is crucial for the understanding of ion-acoustic waves, ion-acoustic turbulence in fusion plasmas and magnetic reconnection in space plasmas.

In the literature, a class of well-established methods for the Vlasov equation is the particle-in-cell (PIC) methods [3,28]. In PIC methods, the macro-particles are advanced in a Lagrangian framework, while the field equations are solved on a mesh. The main advantage of the PIC method is its relatively low cost for high dimensional problems, but it suffers from statistical noise built intrinsically in those methods. In recent years, there has been growing interest in computing kinetic equations in a deterministic fashion, see for example [7,37,13,12,26]. Our approach in this paper is to use a grid-based Vlasov solver, which does not have statistical noise and can resolve the low-density regions more accurately. While there are abundant literature on grid-based Vlasov solver for single-species VA or Vlasov–Poisson (VP) system, e.g. [7,37,4,25,29,17,23], there are relatively fewer published works for the two-species system. In [21,22,20], Fourier transformed methods are used to compute two-species VP system for electron and ion holes. In [31], the hydrodynamic and quasi-neutral limits of the two-species VP system are studied by the finite difference WENO method. In [36,35], the MacCormack method is employed for calculation of anomalous resistivity and the nonlinear evolution of ion-acoustic instability. A detailed study of the comparison of the MacCormack method and a PIC method for anomalous resistivity in current-driven ion-acoustic waves can be found in [38].

One of the main focuses of this paper is to develop fully discrete energy-conserving methods. The total energy is a nonlinear quantity that depends on the distribution functions of both species as well as the electric field. To achieve energy conservation, special care must be taken to design both the temporal and spatial discretizations. In this work, we generalize our previous methods for the single-species VA system [9] and Vlasov–Maxwell (VM) system [8] to the two-species system. We show that the energy-conserving time discretizations for single-species equations [9,8] can also work for the two-species case if extended properly. In particular, for the current-driven ion-acoustic instability, a modification for the implicit method is necessary to fully decouple the split equations. This is achieved by a Gauss–Seidel type iteration technique. The importance of our work also lies in the fact that it can be easily adapted to multi-species VA or VM systems for more complicated physical problems.

We emphasize that the main feature of our methods compared to other high order schemes in the literature is that it can preserve the total energy on the fully discrete level regardless of mesh size. Therefore, even for long time simulation on under-resolved mesh, there will be no generation of annulation of spurious energy, thus avoiding artifacts such as plasma self heating or cooling [16]. Previously, several PIC methods have been proposed to conserve the total energy for the single-species VA system [6] and the VM system [33]. Finite difference and DG methods [24,2] were proposed to conserve the total energy of VP systems on the semi-discrete level. Our method is the first Eulerian solver to achieve conservation of total energy and particle number simultaneously for the two-species system. This is done by using the newly developed energy-conserving temporal discretizations [9,8], and the discontinuous Galerkin (DG) spatial discretizations [14].

For the two-species system, one of the other computational challenges besides conservation is the multiscale nature of the problem. Because ions are much heavier than electrons, electrons move faster and the temporal scale for electrons is smaller than that of the ions. For efficient calculations, hybrid and multiscale particle codes have been developed [5,32]. We mention in particular the implicit particle methods [19,15] and electron sub-cycling techniques [1]. In this paper, we aim at resolving the physical phenomena that happen at the electron time scale. Therefore the typical time step Δt satisfies $w_{pe}\Delta t \propto O(1)$, where w_{pe} is the electron plasma frequency. In the velocity space, we take the common approach of choosing different computational domain for the velocity space of the electrons and ions, and taking larger grids in electrons than ions. This is allowed because the two species are only coupled together through the electric field. We want to remark that in some applications, it would be natural to follow the slower ion time scales. In those scenarios, the electron equation becomes stiff and a multiscale temporal solver would be necessary. However, we do not attempt to address this issue in the current paper and leave it to our future work.

The rest of this paper is organized as follows: in Section 2, we describe the equations under consideration. In Section 3, we develop our energy-conserving schemes and discuss their properties. The additional term involving the spatial average of the current density will cause the split equation to be globally coupled. To resolve this issue, a Gauss–Seidel iteration is employed. Section 4 is devoted to numerical results, including the test of one-species limit and the simulations of the current-driven ion-acoustic waves (CDIAW), in which we perform numerical tests on an ensemble of 100 VA simulations with random phase perturbations to investigate the anomalous resistivity with a reduced mass ratio. Finally, we conclude with a few remarks in Section 5.

2. The two-species VA system

In this section, we describe the two-species VA system and its dimensionless version. The two-species VA system for electrons and ions is given by

$$\partial_t f_\alpha + \mathbf{v}\cdot\nabla_{\mathbf{x}} f_\alpha + \frac{q_\alpha}{m_\alpha}\mathbf{E}\cdot\nabla_{\mathbf{v}} f_\alpha = 0, \quad (\mathbf{x},\mathbf{v}) \in \left(\Lambda_x, \mathbb{R}^n\right) \tag{2.1a}$$

$$\partial_t \mathbf{E} = -\frac{1}{\epsilon_0}(\mathbf{J} - \mathbf{J}_{ext}), \quad \mathbf{x} \in \Lambda_x \tag{2.1b}$$