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A fast nonlinear conjugate gradient based method for 3D concentrated frictional contact problems

Jing Zhao [a,*], Edwin A.H. Vollebregt [a,b], Cornelis W. Oosterlee [a,c]

[a] Delft Institute of Applied Mathematics, Delft University of Technology, Mekelweg 4, 2628CD Delft, The Netherlands
[b] VORtech BV, 2600AG Delft, The Netherlands
[c] CWI – Center for Mathematics and Computer Science, 1090GB Amsterdam, The Netherlands

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ABSTRACT

This paper presents a fast numerical solver for a nonlinear constrained optimization problem, arising from 3D concentrated frictional shift and rolling contact problems with dry Coulomb friction. The solver combines an active set strategy with a nonlinear conjugate gradient method. One novelty is to consider the tractions of each slip element in a polar coordinate system, using azimuth angles as variables instead of conventional traction variables. The new variables are scaled by the diagonal of the underlying Jacobian. The fast Fourier transform (FFT) technique accelerates all matrix–vector products encountered, exploiting the matrix' Toeplitz structure. Numerical tests demonstrate a significant reduction of the computational time compared to existing solvers for concentrated contact problems.

1. Introduction

1.1. Physics

The frictional contact problem has attracted interest from many researchers, due to various applications in the industry and engineering fields, e.g. rolling contact fatigue (RCF) [10], the fatigue life of machine elements [31], friction and wear [7,29,13]. This problem concerns two elastic bodies. When they are pressed together, the forces they obtain from each other result in elastic deformation. This yields a contact area where the surfaces of the two bodies coincide, and exert stresses on each other. These stresses are composed of normal stress (pressure), and the frictional stress (traction) acting in the tangential direction. When and where the frictional stress is small, the two bodies stick to each other. However, local sliding occurs where the frictional stress is large enough. The challenge is to find the distribution of the frictional stress, and the subdivision of the contact area: which part is an adhesion area and in which part does slip occur.

The model for frictional contact starts with a known contact area and pressure distribution. Then the frictional stress should satisfy:

1. In the adhesion area, the magnitude of the tractions does not exceed the traction bound, and there is no slip.
2. In the slip area, the traction bound is reached, and the resulting slip points in the opposite direction of the tractions.

* Corresponding author.
E-mail address: J.zhao-1@tudelft.nl (J. Zhao).

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The traction bound comes from the frictional law that is used. For this one may take Coulomb's law locally, which states that the traction bound equals the product of the normal pressure and a friction coefficient. The magnitude of the tangential tractions should be less or equal to the traction bound. On the one hand, this gives rise to inequality constraints. On the other hand, when slip occurs, equality should hold, and the directions of the tangential tractions and the resulting slip should be opposite. This brings in nonlinearity.

1.2. Solution strategies

The classic solutions to frictional contact problems with partial sliding stem from the work by Cattaneo [4] and Mindlin [23]. In the last decades, other solution techniques have been studied, for example Johnson [14] and Kalker [17] contributed with fundamental work.

The numerical solution techniques employ variational inequalities [14,17,38]. They are generally divided into two classes. One is the class of finite element methods (FEM) [18,38,20,6,2,12], that are widely used, especially in the case of large deformations, and nonlinear elastic materials. These methods typically focus on overall behavior. Due to the discretization of the contacting bodies, this method can be computationally expensive. The other class is the boundary element methods (BEM) [17,1,22,19], that are well-suited for "concentrated contact" and efficient for homogeneous elastic problems. The boundary value problem is transformed to a boundary integral equation. The dimensionality of the problem decreases, i.e., the 3D contact problem is solved by considering 2D contact regions where only the boundary is discretized. Hence, this method reduces the computational time significantly.

When the contacting bodies are of different materials, the tangential tractions and normal displacements interact with each other. In this case, the normal and tangential problems cannot be easily separated. A straightforward way to process it is to solve a fully coupled formulation [37]. Another popular approach is via the so-called "Panagiotopoulos process" [25,9,17]. In each iteration, the normal problem is solved first followed by the tangential problem. When contacting bodies are of the same material, i.e. a so-called quasi-identity case, these two problems can be decoupled, and one iteration is sufficient [17].

1.3. Solution algorithms

Kalker's variational approach [17], which is a prominent method for the rolling contact problem, employs Green's function for the elastic half-space. This is a BEM, where Coulomb's law is applied. A fine discretization is used inside the contact area. Dense matrices need to be solved for elements in the contact area.

As a solution algorithm, the TANG algorithm was proposed in [16]. It applies an active set strategy [24], which leads to systems of nonlinear equations that are solved using Newton's method and Gauss-elimination (GE). This approach has $\mathcal{O}(n^{3.5})$ complexity, with n the number of contact elements. Another method is the ConvexGS method [32]. It reduces the global problem to a small-sized optimization problem on each element, and solves by a block Gauss–Seidel iteration. This method is incorporated into the software CONTACT [35]. However, the Gauss–Seidel process is also relatively slow for fine discretizations with a complexity of about $\mathcal{O}(n^{2.3})$ [33].

Different from BEM, FEM is based on a large number of elements covering whole contacting bodies, while much fewer elements are placed in the contact area. Sparse matrices are solved, but the size of the matrices is much larger than the dense matrices involved in BEM. Algorithms include the penalty approach, the augmented Lagrangian technique, etc. Comparing with the BEM methods, we encounter similar approaches for the nonlinear equations, like Newton-based methods [38], or a nonlinear Gauss–Seidel method [15] that is similar to ConvexGS.

1.4. A new solution method

The motivation of our work is to develop a fast solver for the 3D frictional contact problem, especially for the so-called shift problem, e.g. the Cattaneo shift [4]. It is a transient contact problem, and concerns one object pressed onto another, and shifted tangentially. It plays an important role in the study of rolling contact problems, since there can be generally a sequence of shift problems. In this paper we consider the tangential problem, with the solutions from the normal problem already available. It can be easily incorporated into the Panagiotopoulos process, to deal with more complicated contact.

Our new method contributes to the BEM solvers. We call it "TangCG", since it searches for the tangential tractions and is based on the nonlinear conjugate gradient method. The constraint that the magnitude of tractions on each slip element should equal the traction bound, inspires to place the traction vector at a circle in a polar coordinate system, with the radius being the traction bound. We use azimuth angles as variables in the slip area, which is a significant difference from conventional solvers.

The TangCG algorithm is a so-called bound-constrained conjugate gradient (BCCG) method, which was proposed for linear complementarity problems in normal contact [34]. The BCCG method uses an active set strategy, and employs the conjugate gradient (CG) method for the governing linear system. Differently, the governing system in frictional contact problems is mainly nonlinear, hence, we employ a nonlinear conjugate gradient (NLCG) method [28]. The TangCG algorithm is combined with a diagonal scaling preconditioner.