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High-order finite-volume methods for hyperbolic conservation laws on mapped multiblock grids

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ABSTRACT

We present an approach to solving hyperbolic conservation laws by finite-volume methods on mapped multiblock grids, extending the approach of Colella, Dorr, Hittinger, and Martin (2011) [10] for grids with a single mapping. We consider mapped multiblock domains for mappings that are conforming at inter-block boundaries. By using a smooth continuation of the mapping into ghost cells surrounding a block, we reduce the inter-block communication problem to finding an accurate, robust interpolation into these ghost cells from neighboring blocks. We demonstrate fourth-order accuracy for the advection equation for multiblock coordinate systems in two and three dimensions.

1. Introduction

The solution of partial differential equations using structured-grid-based discretizations can be challenging when the solution domain has significant geometric structure or is more easily expressed in non-Cartesian coordinates. For instance, in the simulation of the plasma near the edge of a tokamak fusion reactor, coordinates defined by the magnetic field are advantageous. As shown in Fig. 1(a), the *single-null* topology of the magnetic field in the edge region [33,42] (shown in a poloidal cross-section) possesses both open and closed field lines separated by a *separatrix* – a flux surface that is self-intersecting. There is no simple mapping of a single rectangular domain to this edge geometry.

Other examples of solution domains that are more easily expressed in non-Cartesian coordinates include the interior of a star or planet and the atmosphere, which is effectively a thin shell over a spherical surface. Although spherical coordinates can be used for both of these cases, they pose difficulties because of the singularities at the center and at the poles.

While mapped-grid approaches based on a single, rectangular Cartesian mesh have the advantage of simplicity and regular access patterns due to the mesh structure, these approaches are extremely limited in the types of domains they can represent well. In contrast, fully unstructured approaches can more easily represent complex geometry, but these require additional storage of mesh associativity data. A popular alternative is to use multiblock meshes (also known as composite patches or zonal grids), where the domain is decomposed into multiple sub-domains that each map to a rectangular block. The complicated tokamak edge geometry, for instance, can be mapped to eight rectangular subdomains that connect at the intersection point of the separatrix, the *X-Point*, as shown in Fig. 1(b).

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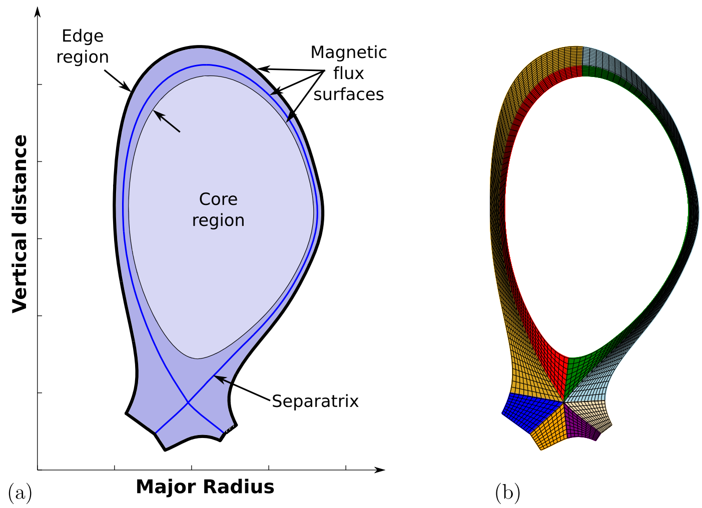

Fig. 1. Poloidal cross section of the single-null magnetic field geometry in a tokamak fusion reactor showing (a) the edge and core regions and the separatrix that separates open and closed magnetic field lines and (b) the decomposition of the single-null domain into eight blocks.

Mapped multiblock grids and, more generally, composite grids (including overset and patch-based refinement) have been used in the solution of partial differential equations (PDEs) since the 1970s [22,34]. A substantial amount of development was done in the computational aerodynamics community for external flows around complex bodies. The Cubed Sphere [31] is a type of multiblock grid that has also been developed for solving PDEs on a spherical surface; in [39], this grid is used with a high-order finite-volume method to solve the shallow-water equations. There is a rich literature on the subjects of mapped and multiblock grids that is too extensive to summarize here; we refer the interested reader to several review articles [2,35,37].

The starting point for the present work is the high-order finite-volume method in Colella et al. [10]. The advantage of this approach is that it is strongly conservative in the sense of [40,41], high-order accurate, and freestream-preserving. It also has the advantage of using a smoothly-varying structured grid for its underlying discretization of space. Discretizations on such grids preserve many of the desirable properties of discretizations on Cartesian grids, such as cancellation of error in centered differences, and relatively simple quadrature rules for computing averages over cells and faces. We extend this method to the case of mapped multiblock grids, in which the computational domain in physical space is represented as the disjoint union of images of mappings that are *conforming*, meaning that they are aligned at common boundaries in such a way that when the maps are discretized, the individual faces of control volumes at those boundaries coincide. To maintain the mapped-grid formalism constraint that mappings are sufficiently differentiable, we define local mappings for each block that, beyond being conforming, need not coincide in any other way. By using a smooth continuation of each mapping beyond its block boundary, we reduce the problem of inter-block communication to that of the accurate interpolation of solution values from neighboring blocks into the halo regions.

Interpolation between neighboring grids is a common problem in multiblock, overset, and patch- and block-based adaptive mesh refinement (AMR) methods. A variety of polynomial interpolation techniques on both solution values and interface fluxes have been developed [9,29,30,32]. A major concern has been interpolation procedures that ensure conservation [6,9,29,30] and stability [5,28,29]. Here, since the blocks share only a lower-dimensional interface (the PDEs are not solved in the halo regions), conservation is easily ensured by using consistent interface fluxes on the block boundaries. The main challenge, instead, is identifying a suitable stencil over which to interpolate. As in overset or AMR techniques, the halo extensions beyond a block may overlap multiple blocks, particularly in the vicinity of mesh singularities. Identifying a suitable collection of cells from the original block and its neighbors is therefore not trivial. In the fully unstructured and "mesh-free" computational-fluid-dynamics literature, one technique for reconstruction is least-squares interpolation [3,4,8,15,19,23,24,27], which does not presume any underlying spatial relationship between the values used in the interpolation. This is the approach we take here. The K-exact reconstruction of Barth [4] uses averages on a selected number of neighboring cells to reconstruct a polynomial that reproduces exactly polynomials of degree up to K and preserves the average value within the computational cell, but our procedure, which is used to find that average value within the computational cell, is not required to be K-exact.

Although less common for structured grids, least-squares reconstruction is intrinsic to the genuinely multi-dimensional, high-order, Central Essentially Non-Oscillatory (CENO) finite-volume schemes [16–18,36,38] that have been successfully applied in 2D and 3D to inviscid and viscous compressible flow, reacting turbulent flow, and ideal magnetohydrodynamics on body-fitted, multiblock grids with block-based adaptive mesh refinement. In particular, the CENO approach has been applied to a block-adaptive cubed-sphere grid [17,18], where the least-squares reconstruction in the flux calculation produces a uniformly high-order solution, even at points of reduced connectivity. In contrast, our use of least-squares interpolation