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A multiple-time-step technique for coupled free flow and porous medium systems

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ABSTRACT

A mass conservative multirate time integration method is developed for a time-dependent isothermal single-phase free flow system coupled with an isothermal single-fluid-phase porous medium system. Long time stability of the proposed numerical scheme is proved under a time step restriction which depends on the physical parameters of the flow systems and the ratio between the time steps applied in the free flow and porous medium domains. Convergence analysis is carried out and *a priori* error estimates are obtained. Numerical results are presented for two model problems and a realistic setup. Advantages of using multiple-time-step techniques for modeling coupled systems where the processes evolve on different time scales are demonstrated and discussed.

1. Introduction

Coupled free flow and porous medium flow systems arise in a wide spectrum of environmental settings and industrial applications (evaporation from the soil influenced by the wind, overland flow interactions with groundwater aquifers, filtration processes, fuel cells). In these flow domains the processes evolve on different scales in space and time that requires different models for each flow system and an accurate treatment of transitions between them at the interface.

In the free flow region, the Navier–Stokes or Stokes equations are typically used to describe momentum conservation while Darcy's law is applied as an approximate conservation of momentum equation in the porous medium. The Beavers–Joseph velocity jump condition [1] is a common practice to couple the two different flow domains, in conjunction with restrictions that arise due to mass conservation and balance of normal forces across the interface. The Beavers–Joseph condition establishes the connection between the free flow velocity and the porous medium velocity tangent to the interface, and it is an additional condition to couple equations of different orders. Saffman proposed a modification of the Beavers–Joseph condition that contains only variables in the free flow region, since the porous medium velocity is much smaller than the free flow velocity, and can thus be neglected [2]. Mathematical justification of the Beavers–Joseph–Saffman interface condition was provided by Jäger and Mikelić using a homogenization technique [3].

Modeling the coupling between free flow and porous medium domains can be done through the sharp interface approach by imposing an appropriate set of interface conditions at the boundary between the flow domains [4–9] or by considering a transition zone between the two flow regions and developing a transition region model [10].

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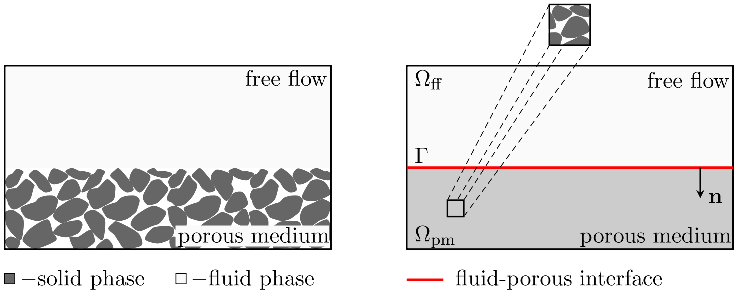

Fig. 1. Schematic representation of the coupled system at the microscale (left) and macroscale (right).

For the last ten years, work has been carried out mainly for stationary problems aimed at providing a rigorous problem formulation and numerical methods for solving such coupled flow systems [4–8]. Recent advances in coupling techniques for nonstationary problems are presented in [7,9,11–16], where the same time step is used in both flow domains.

The fluid velocity in the free flow domain is usually much higher than in the porous medium, therefore it is reasonable to apply a multiple-time-step technique, i.e., to compute fast solutions using a small time step and consider a larger time step for slow solutions. First results on the multirate time integration technique for coupled time-dependent Stokes equations and Darcy's law are presented in [17], where a decoupled scheme is proposed and stability analysis is provided. A multiple-time-stepping strategy for shallow water equations coupled with Richards' equation is proposed in [18]. Multirate time integration methods for other applications can be found in [19–22].

The overall goal of this work is to develop and investigate a mass conservative multiple-time-step method for solving coupled free flow and porous medium flow problems. We present a numerical algorithm different from the one given in [17] so that the free flow velocity is not averaged at the interface, and prove long time stability of the method. The time step restriction depends in our case not only on the model parameters, as in [17], but also includes the ratio between the time steps applied in the free flow and porous medium domains. In addition, the proposed decoupled scheme is locally mass conservative. Also, *a priori* error estimates are obtained. Numerical results which demonstrate the efficiency of the proposed algorithm are presented for a model problem, a model problem with realistic coefficients and a realistic setup.

The paper is organized as follows. In Section 2 we describe the flow system of interest which includes two flow models applied in the free flow and porous medium domains and the boundary conditions at the fluid-porous interface. Section 3 is devoted to the weak formulation for the coupled problem. A multiple-time-stepping technique is described in Section 4, and analysis of the proposed algorithm is provided in Section 5. Numerical experiments are presented in Section 6. Finally, we discuss advantages of the multiple-time-step strategy for modeling multiphysical systems with the processes running on different time scales and comment on possible extensions of this work.

2. Flow system description

The system of interest includes a free flow region $\Omega_{\rm ff}$, containing a single fluid phase, and a porous medium layer $\Omega_{\rm pm}$, which contains a fluid and a solid phase. At the microscale (pore scale), the flow in the entire fluid domain (the free flow region plus the pores in the porous medium) can be described by the Navier–Stokes equations with a no-slip condition on the velocity prescribed at the boundaries between fluid and solid (Fig. 1, left). Nevertheless, the computation of the microscale flow field throughout the fluid phase is infeasible for many practical applications because it requires detailed information about the porous medium morphology and topography which is usually unknown. At the macroscale (Darcy's scale), the system is described as two different continuum flow domains (free flow, porous medium) separated by the interface Γ (Fig. 1, right). It can be a sharp interface [4,7] or a transition region of a positive thickness [10].

To describe the coupled system, the two flow models applied in the free flow and the porous medium domains and an appropriate set of interface conditions should be formulated. In the paper, the sharp interface concept is applied for coupling the two flow domains. This approach does not assume any storage and/or transfer of mass, momentum and energy along the interface, and the boundary conditions at the fluid-porous interface are algebraic jump conditions.

We consider the same fluid in the free flow and porous medium domains. In this case, water provides most of the applications, e.g., an overland flow (lake, river, wetland) interacting with a saturated groundwater system. Since it is reasonable to apply multiple-time-step techniques only for coupled systems where the processes evolve on different time scales, the primary application in this work is chosen to be the river interaction with groundwater aquifers. We deal with isothermal processes and assume the fluid to be incompressible.

2.1. Free flow model

In the free flow domain $\Omega_{\rm ff}$, the flow is commonly modeled by the Navier–Stokes equations. Considering laminar flows and neglecting the inertia term, the momentum balance reduces to the Stokes equation