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Efficient high order waveguide mode solvers based on boundary integral equations ☆

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ABSTRACT

For optical waveguides with high index contrast and sharp corners, high order full-vectorial mode solvers are difficult to develop, due to the field singularities at the corners. A recently developed method (the so-called BIE-NtD method) based on boundary integral equations (BIEs) and Neumann-to-Dirichlet (NtD) maps achieves high order of accuracy for dielectric waveguides. In this paper, we develop two new BIE mode solvers, including an improved version of the BIE-NtD method and a new BIE-DtN method based on Dirichlet-to-Neumann (DtN) maps. For homogeneous domains with sharp corners, we propose better BIEs to compute the DtN and NtD maps, and new kernel-splitting techniques to discretize hypersingular operators. Numerical results indicate that the new methods are more efficient and more accurate, and work very well for metallic waveguides and waveguides with extended mode profiles.

1. Introduction

Optical waveguides [1–3] are structures that can guide the propagation of light. They are widely used as basic components in integrated optical circuits and optical communication systems. In recent years, many complicated optical waveguides have appeared, such as photonic crystal fibers [4], plasmonic waveguides [5], etc. These new waveguides have attracted much attention due to their unique abilities in confining light. For an optical waveguide, the most important mathematical problem is the computation of waveguide modes. For a waveguide which is invariant along its axis z, a guided mode is a special solution of Maxwell's equations that depends on z as $\exp(i\beta z)$ and decays exponentially away from the waveguide core, where β is the so-called propagation constant. Open waveguides also have leaky modes which exhibit outgoing wave behavior away from the waveguide core. Throughout this paper, we consider only guided modes.

Classical optical fibers can be studied using a scalar model, since the refractive indices of the core and the cladding are nearly equal. There are also semi-vectorial models that are applicable to some waveguides. For waveguides with high index contrast, such as silicon waveguides, plasmonic waveguides and photonic crystal fibers, full-vectorial methods are necessary. Currently, there exist many different full-vectorial mode solvers, including the finite difference method [6–14], the finite element method [15–22], the multi-domain pseudospectral method [23–25], etc. However, for waveguides with

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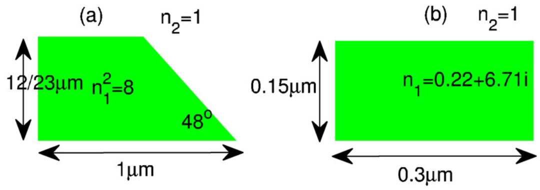

Fig. 1. Waveguides with a single core surrounded by a homogeneous medium: (a) a dielectric waveguide with a trapezoidal core; (b) a metallic waveguide with a rectangular core.

sharp corners, it is very difficult to find any high order numerical method, since the electromagnetic field may be singular at the corners.

Boundary integral equation (BIE) methods have been used to analyze optical waveguides [26–33]. They are highly competitive, since they can easily handle general refractive-index discontinuities (i.e., interfaces), discretize on the interfaces only, and give rise to small matrices. Existing BIE methods reported in [26–28] exhibit low convergence orders due to their use of boundary elements. For waveguides with smooth interfaces, high order BIE methods with exponential convergence are available [29–31]. However, these high order methods need to solve four functions on each interface. In a recent work [32], we developed a high order BIE method that solves only two functions on each interface. Our method also has exponential convergence, and is almost eight times faster than other BIE methods. We call this method BIE-DtN method, since it relies on the Dirichlet-to-Neumann (DtN) map for each homogeneous domain (with a constant refractive index). However, all these high-order BIE methods encounter difficulties when the waveguide has sharp corners, since the methods used in these papers for discretizing boundary integral operators could fail. In [33], we developed a high order full-vectorial BIE mode solver for waveguides with sharp corners. We call this method the BIE-NtD method, since it relies on the Neumann-to-Dirichlet (NtD) map for each homogeneous domain. The method achieves high order convergence for dielectric waveguides with corners, but it needs to solve four functions on each interface.

In this paper, we develop new versions for both BIE-DtN and BIE-NtD methods. The new BIE-DtN method still solves two functions on each interface, but now handles waveguides with sharp corners. The DtN map for a general domain with corners is computed using a BIE with a hypersingular integral operator and extra terms corresponding to the corners. For the hypersingular integral operator on a smooth boundary, Kress [34] developed a high-order kernel-splitting technique for its discretization, but the method fails on boundaries with corners. We develop a new kernel-splitting technique to overcome this difficulty. Our new BIE-NtD method still handles waveguides with sharp corners, but solves three (instead of four) functions on each interface. Although it still solves one more function than the BIE-DtN method, the BIE-NtD method is simpler to implement. Furthermore, many optical waveguides have interfaces extending to infinity, leading to domains of constant refractive index with unbounded boundaries. We also develop well-approximated BIEs to compute both DtN and NtD maps for these domains. Overall, these new BIE mode solvers are more general and solve minimum number of unknowns on each interface. As illustrated by the numerical examples, these new methods bring a large saving in the computational cost and a significant improvement in the accuracy.

2. Problem formulations

To illustrate the basic ideas clearly, we start with a simple case where the optical waveguide involves a finite core surrounded by a homogeneous medium (the cladding) as shown in Fig. 1. The waveguide structure is invariant in the z-direction, and its cross-section in the xy-plane consists of two homogeneous domains, a bounded domain Ω_1 with refractive index n_1 and an unbounded domain Ω_2 with refractive index n_2, which share a common boundary Γ. Both the core and the cladding are assumed to be non-magnetic. For dielectric waveguides, the refractive indices satisfy $n_1 > n_2$. We also consider metallic waveguides for which n_1 is complex. Here $\{x, y, z\}$ is the standard Cartesian coordinate system. In the following, we only consider homogeneous domains with Lipschitz and piecewise smooth boundaries. For time harmonic waves with the time dependence $\exp(-i\omega t)$ where ω is the angular frequency, the governing Maxwell's equations are

$$\nabla \times \boldsymbol{E} = ik_0 \boldsymbol{H}, \tag{1}$$

$$\nabla \times \boldsymbol{H} = -ik_0 \varepsilon \boldsymbol{E}. \tag{2}$$

In the above, ∇ denotes the gradient operator, $\boldsymbol{E}$ is the electric field, $\boldsymbol{H}$ is the magnetic field multiplied by the freespace impedance, $k_0 = \omega/c = 2\pi/\lambda$ is the freespace wavenumber, c is the speed of light in vacuum, λ is the free space wavelength, $\varepsilon = n^2$ is the dielectric function, and it is piecewise constant and independent of z. On the interface Γ, the four components E_z, H_z, H_x and H_y are continuous where E_z denotes the z-component of $\boldsymbol{E}$, etc.

A guided mode of the waveguide is a special solution of Eqs. (1) and (2) such that both $\boldsymbol{E}$ and $\boldsymbol{H}$ depend on z as $\exp(i\beta z)$ and decay exponentially to zero as $\sqrt{x^2 + y^2} \to \infty$, where β is called the propagation constant. To find β, BIE methods typically work on only two components: $\{E_z, H_z\}$ or $\{H_x, H_y\}$. For waveguides with non-magnetic media and domains with corners, we prefer to use $\{H_x, H_y\}$, since they are smoother than other components, and finite even at corners. In each domain Ω_j, both H_x and H_y satisfy the following Helmholtz equation