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A simple Cartesian scheme for compressible multimaterials

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ABSTRACT

We present a simple numerical method to simulate the interaction of two non-miscible compressible materials separated by an interface. The media considered may have significantly different physical properties and constitutive laws, describing for example fluids or hyperelastic solids. The model is fully Eulerian and the scheme is the same for all materials. We show stiff numerical illustrations in case of gas–gas, gas–water, gas–elastic solid interactions in the large deformation regime.

1. Introduction

Physical phenomena that involve several materials are ubiquitous in nature and applications: multiphase flows, fluid–structure interaction, impacts, to cite just a few examples. In recent literature several strategies were proposed to attack these problems: Lagrangian models [29], Arbitrary-Lagrangian–Eulerian (ALE) models [11,20], Eulerian models [4,9,19,23]. In this context, immersed boundary methods [17,21] are an option to discretize boundary conditions at the interface, trading off accuracy for mesh generation simplicity. The literature in this domain is vast and each of the approaches cited include a variety of methods that are adapted to specific applications.

Broadly speaking Lagrangian methods are interesting because the interface between the materials is fixed in the reference domain and a body-fitted mesh is generated once for all. The interface equilibrium conditions can be taken into account in a simple and accurate way. Free-surface problems and purely elastic problems are the paradigmatic applications of this approach. On the other hand, if one of the materials is a fluid, the mapping from the physical to the reference domain can become very irregular or even singular for large times. ALE methods take into account this peculiarity of fluid flows by discretizing the physical domain over an unsteady mesh that can also cope with the moving interfaces. Moreover, the mapping from the physical domain to the computational domain is independent of the trajectories of the fluid particles and therefore its regularity can be kept under control. The counterpart is a more complex scheme formulation and numerical implementation. Also, when large deformations occur, mesh generation and partitioning can pose challenging problems. In Eulerian methods the problem is approximated on a fixed mesh in the physical domain, but additional modeling is required to obtain a consistent, stable and accurate description of the (eventually smoothed) material interface evolution.

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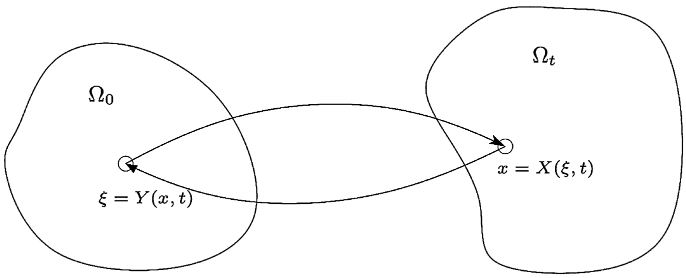

Fig. 1. Forward and backward characteristics.

Immersed boundary methods can be used to keep the material interface sharp. With this approach, the contact discontinuity can arbitrarily cross the grid and the transmission conditions, usually in terms of velocity and stress, are applied via interpolation. The models considered on either side of the interface can be either Lagrangian or Eulerian, using Lagrangian markers or level-set functions to describe the interface according to the specific problem considered.

In this study, we are interested in the numerical simulation of phenomena such as the scattering of shock waves at gas–water interfaces, the propagation of non-linear elastic waves from a hyperelastic solid to a fluid and vice versa, the scattering of non-linear elastic waves at solid–solid material discontinuities. These phenomena can be modeled by a fully Eulerian system of conservation laws that applies to every material; only the constitutive law may change, reproducing the mechanical characteristics of the medium under consideration. For example, an elastic material or a gas will be modeled by the same set of quasi-linear hyperbolic partial differential equations (PDEs) except for the constitutive law relating the material deformation and the stress tensor.

The systematic derivation of such models starting from continuum mechanics principles, their thermodynamic consistency and the corresponding wave-propagation patterns were initially studied in [13]. Their numerical simulation is delicate because standard Godunov schemes lead to pressure oscillations at the material contact discontinuity already in the case of multifluids. In [1] the pressure perturbation mechanism at the origin of this phenomenon was explained and a first fix was proposed. An effective remedy to this problem was presented in [10] with the ghost-fluid method (sharp interface between the materials). For multifluids, improvements of this approach requiring less storage were proposed in [2] (diffuse interface) and [8] (sharp interface). The common idea of these methods is to define a "ghost" fluid that has continuous mechanical characteristics across the interface, but the same thermodynamic state or the same equation of state of the actual fluid. This assumption leads to locally non-conservative schemes that are consistent, stable and non-oscillatory at the material interface.

For elastic compressible materials existing methods either rely on the definition of ghost materials (see [4,23] for hyperelastic models and [19,27] for a hypoelastic formulation) or on mixture models and diffuse interfaces [9]. In this paper we propose a simple, stable and non-oscillatory scheme for multimaterials that avoids the definition of a ghost hyperelastic medium. The equilibrium boundary conditions at the material interface are imposed like in immersed boundary methods, in the same spirit of what is done in [14] for rigid bodies. Therefore, in our scheme the evolution of the material discontinuity is sharp by construction. In this paper we limit the discussion to the two dimensional case but the models and the numerical scheme can be extended in 3D.

In the following we detail the Eulerian model, the numerical method and we present a set of stiff test-cases involving fluid and hyperelastic compressible materials.

2. The model

This model was already discussed in [6,9,13,23–25]. In this section, we develop the principal elements of the formulation.

2.1. Forward and backward characteristics

Let $\Omega_0 \subset \mathbb{R}^2$ be the reference or initial configuration of a continuous medium and $\Omega_t \subset \mathbb{R}^2$ the deformed configuration at time t. In order to describe the evolution of this medium in the Lagrangian frame we define the forward characteristics $X(\xi, t)$ as the image at time t in the deformed configuration of a material point ξ belonging to the initial configuration, i.e., $X : \Omega_0 \times [0, T] \to \Omega_t$, $(\xi, t) \mapsto X(\xi, t)$ (see Fig. 1). The corresponding Eulerian velocity field u is defined as $u : \Omega_t \times [0, T] \to \mathbb{R}^2$, $(x, t) \mapsto u(x, t)$ where

$$\begin{cases} X_t(\xi, t) = u\big(X(\xi, t), t\big) \\ X(\xi, 0) = \xi \end{cases} \tag{1}$$

To describe the continuous medium in the Eulerian frame, we introduce the backward characteristics $Y(x, t)$ (see [6]) that for a time t and a point x in the deformed configuration, gives the corresponding initial point ξ in the initial configuration,