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An arbitrary Lagrangian–Eulerian formulation for the numerical simulation of flow patterns generated by the hydromedusa *Aequorea victoria*

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A B S T R A C T

A new geometrically conservative arbitrary Lagrangian–Eulerian (ALE) formulation is presented for the moving boundary problems in the swirl-free cylindrical coordinates. The governing equations are multiplied with the radial distance and integrated over arbitrary moving Lagrangian–Eulerian quadrilateral elements. Therefore, the continuity and the geometric conservation equations take very simple form similar to those of the Cartesian coordinates. The continuity equation is satisfied exactly within each element and a special attention is given to satisfy the geometric conservation law (GCL) at the discrete level. The equation of motion of a deforming body is solved in addition to the Navier–Stokes equations in a fully-coupled form. The mesh deformation is achieved by solving the linear elasticity equation at each time level while avoiding remeshing in order to enhance numerical robustness. The resulting algebraic linear systems are solved using an ILU(k) preconditioned GMRES method provided by the PETSc library. The present ALE method is validated for the steady and oscillatory flow around a sphere in a cylindrical tube and applied to the investigation of the flow patterns around a free-swimming hydromedusa *Aequorea victoria* (crystal jellyfish). The calculations for the hydromedusa indicate the shed of the opposite signed vortex rings very close to each other and the formation of large induced velocities along the line of interaction while the ring vortices moving away from the hydromedusa. In addition, the propulsion efficiency of the free-swimming hydromedusa is computed and its value is compared with values from the literature for several other species.

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1. Introduction

Moving boundary problems in computational fluid dynamics have become of great interest due to their wide range of application areas. Examples include a wide variety of fluid-structure interaction problems such as wing flutter and tail buffeting, a large class of free-surface problems, fluid-particle interactions, swimming/flying animals, rotor-fuselage aerodynamic interactions, etc. In order to simulate the flow problems with moving boundaries, several numerical approaches have been presented in the literature including the arbitrary Lagrangian–Eulerian (ALE) method [20], the immersed boundary method [37,32] and the fictitious domain method [18].

In the ALE method, the mesh follows the interface between the fluid and solid boundary and the governing equations are discretized on an unstructured moving mesh. This differs from the standard Eulerian formulation in a way that the mesh movement has to fulfill special conditions in order to maintain the accuracy and the stability of the time integration scheme. This condition is satisfied by the enforcement of the so-called geometric conservation law (GCL) as coined by Thomas and

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Lombard [48]. The geometric conservation law requires that the volumetric increment of a moving cell must be equal to the summation of the volumes swept by its surfaces that close the volume. It can be interpreted such that a numerical scheme should preserve a uniform flow solution exactly independent of the mesh motion. Although the GCL is satisfied easily in the continuous sense, their discrete implementation may not be trivially satisfied. The ALE time integration scheme developed by Koobus and Farhat [26] is based on more continuous time integration of the fluxes. Such a scheme offers second-order accuracy in time obeying the GCL, but the integration will be computationally expensive. Geuzaine et al. [17] have showed that the GCL is neither a necessary nor a sufficient condition condition for an ALE scheme to preserve its order of time-accuracy established on fixed meshes. Recently, Mavriplis and Yang [29] have proposed a general framework for deriving high-order temporal schemes which respects the GCL. In the present work, a geometrically conservative arbitrary Lagrangian–Eulerian formulation is presented in the swirl-free cylindrical coordinates. The governing equations are multiplied by the radial distance r so that the GCL takes very simple form similar to that of the Cartesian coordinates. In addition, this will allow us to avoid singularities related to $1/r$ and $1/r^2$ terms in the cylindrical coordinates and result in better conditioned linear systems. Although similar modifications to the Navier–Stokes equations are presented in the literature [21], they are not considered in the view of the GCL.

The modified governing equations are discretized using the semi-staggered finite volume method [41,43] on all-quadrilateral unstructured moving meshes while allowing to the use of structured meshes as well. The continuity equation is satisfied exactly within each element and a special attention is given to satisfy the geometric conservation law at discrete level. The choice of the present semi-staggered approach leads to better pressure coupling compared to non-staggered (collocated) approach while being capable of handling non-Cartesian grids. In addition, it eliminates the need for a pressure boundary condition since it is defined at interior points. Furthermore, the summation of the continuity equation within each element can be exactly reduced to the domain boundary, which is important for the global mass conservation. But the most appealing feature of the method is leading to very simple algorithm consistent with the boundary and initial conditions required by the Navier–Stokes equations. Recently, the semi-staggered arrangement of variables has been used by Rida et al. [38], Kobayashi et al. [25] and Wright and Smith [50] for triangular, quadrilateral and hybrid meshes in 2D. The extension of the semi-staggered approximation to arbitrary Lagrangian–Eulerian form is reported by Hirt et al. [20] and Smith and Wright [47]. The convective fluxes in the momentum equations are approximated using both the second-order simple averages and the second-order upwind least square interpolation [2,4] in order to maintain stability at higher Reynolds numbers. The mesh motion is determined by solving the linear elasticity equation similar to the work of Refs. [22,14] at each time level while avoiding remeshing in order to enhance numerical robustness.

The resulting algebraic linear systems are solved using the GMRES method [40] with the restricted additive Schwarz preconditioner. The implementation of the preconditioned Krylov subspace algorithm and the restricted additive Schwarz preconditioner were carried out using the PETSc [3] software package developed at the Sandia National Laboratories. The computational meshes are partitioned using the METIS library [24] and within each sub-domain incomplete LU (ILU(k)) preconditioner is employed. In order to avoid the zero-block in the saddle point problem, we use an upper triangular right preconditioner which results in a scaled discrete Laplacian instead of a zero block in the original system. Unfortunately, this leads to a significant increase in the number of non-zero elements following the matrix-matrix multiplication. However, the new system may be efficiently preconditioned using ILU(k) preconditioner. The algebraic linear systems are solved in a fully-coupled manner including the equation of motion of a deforming body. This will lead to more robust solution techniques compared to SIMPLE, SIMPLER, etc. type decoupled solution techniques. Convergence of these decoupled solution techniques can often be problematic and may even result in nonconvergence. An extensive review on the fully-coupled iterative solvers for the incompressible Navier–Stokes equations may be found in [44].

The present ALE formulation is applied to the free-swimming oblate hydromedusa *Aequorea victoria* (crystal jellyfish) in order to investigate the propulsion mechanism of an oblate hydromedusa. The experimental observations indicate that the oblate medusae create more complex wake structure than those observed of more prolate jetting medusa and swim with jet-paddling mode of propulsion [30]. However, this propulsion mechanism for the oblate medusae is not well understood in the literature. The present numerical simulations for the free-swimming oblate hydromedusa *A. victoria* indicate the shed of the opposite signed vortex rings very close to each other and the formation of large induced velocities along the line of interaction while the ring vortices moving away from the hydromedusa. This mechanism of propulsion is very similar to the experimental observations of Dabiri et al. [12] for the hydromedusa *Aurelia aurita*. Although Ford et al. [15] have reported series of toroid vortex rings traveling along the medusan oral arms and tentacles, which is very similar to our fixed medusa simulations, this is particularly due to its very large tentacles causing the hydromedusa *Chrysaora quinquecirrha* hardly move through the surrounding fluid.

The remainder of this paper is organized as follows: In Section 2 the present ALE method is described along with the geometric conservation law and the mesh deformation technique. Validation of the present numerical method is given in Section 3. This is followed by the discussion on the flow patterns generated by the free swimming of the hydromedusa *A. victoria*. Conclusions are provided in Section 4.

2. Mathematical and numerical formulation

The incompressible Navier–Stokes equations that govern the swirl-free axisymmetric viscous fluid flow in the Eulerian cylindrical coordinates system (x, r) can be written in dimensionless form as follows: the continuity equation