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Vorticity–divergence mass-conserving semi-Lagrangian shallow-water model using the reduced grid on the sphere

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ABSTRACT

The semi-Lagrangian semi-implicit shallow water model on the sphere using the reduced latitude–longitude grid is presented. The key feature of the model is the vorticity–divergence formulation on the unstaggered grid. The new algorithm for the reconstruction of wind components from vorticity and divergence is described. The mass-conservative version of the model is developed. The conservative cascade scheme (CCS) by Nair et al. is modified to provide a locally-conservative semi-Lagrangian advection algorithm for the reduced grid. Some numerical advection tests are carried out to demonstrate the accuracy of the CCS with the reduced grid. The CCS-based discretization for the continuity equation and finite-volume Helmholtz problem solver are introduced to guarantee the mass-conservation.

The results for shallow water tests on the sphere are presented. The results for different versions of the model are compared. They are also compared with the results for the same tests available in literature. The impact of the reduced grid is analyzed. The mass-conservative version of the model using the reduced grid with up to 20% reduction of grid points number has approximately the same accuracy as its non-conservative counterpart implemented on the regular latitude–longitude grid.

1. Introduction

The algorithms for discretization of the atmospheric equations on the sphere attract continuous interest. Many alternatives are considered in the literature: spectral element method [1–3], pseudo-spectral method [4], double Fourier series [5,6], finite-element methods on the icosahedral grids [7], discontinuous Galerkin method [8,9], finite-volume methods on the cubed sphere [10,11], icosahedral grid [12–14], and Yin-Yang grid [15]. Recently, flexible methods have been designed for arbitrary unstructured grids [16].

The semi-Lagrangian advection [17] coupled to the semi-implicit time integration scheme allows to use large time step in an atmospheric model, so there are many global and regional atmospheric models used for operational numerical weather prediction which are based on this approach: IFS/ARPEGE global model (dynamical core used in ECMWF and Météo-France) [18], UK Unified Model [19] (also used in Australia and South Korea), GEM global model at Canadian Meteorological Center [21], Japan Meteorological Agency global model [20], limited-area models ALADIN [22] (consortium of 16 countries), HIRLAM [23] (consortium of 10 countries), MC2 [24] (Canada). The SL-AV (Semi-Lagrangian Absolute-Vorticity) model is a 3D version of the shallow-water vorticity–divergence semi-Lagrangian model on the sphere presented in [25]. The SL-AV model [26] has become operational weather forecast model at the Hydrometcentre of Russia. The shallow-water and 3D versions of the model [25] with variable-resolution in latitude were also successfully implemented [27].

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However, now there are new challenges related to future implementation of the atmospheric models with the horizontal resolution of order 1 km. First of all, the regular latitude–longitude grid used in the SL-AV model will pose a serious problem due to the concentration of grid points near the poles. This drawback leads to the problems in use of parallel iterative solvers, calculation of grid-point humidity convergence needed for deep convection parameterization. Several alternative grids were proposed in the literature: icosahedral grid, cubed sphere, Yin-Yang grid. The early review of various grids used in atmospheric models can be found in [28], and some applications of these grids are described for example in [29–31]. Note also the recent review article [32]. At the same time, it is easier to construct a high-order numerical approximation to governing equations on the latitude–longitude grid. It is also easier to code the resulting algorithm. The disadvantages of a regular latitude–longitude grid can be to a significant extent overcome with the use of the properly constructed reduced latitude–longitude grid. The review on application of the reduced grid is presented in Section 2.

A modern atmospheric model, both climate and numerical weather prediction model, includes ozone and small gas constituents description. This ultimately requires local mass conservation in numerical algorithms for 3D advection (transport). This constitutes a second challenge for a semi-Lagrangian model.

There is a number of papers devoted to the mass-conserving semi-Lagrangian transport algorithm [33–36,30], the latter using the cubed sphere. The finite-volume based global semi-Lagrangian transport schemes [34,35] on the regular latitude–longitude grid make use of the computationally efficient dimension splitting approach [37]. In [38], the semi-implicit semi-Lagrangian limited area shallow water model on the sphere is constructed with a mass-conserving transport scheme. The work [39] presents a mass-conserving semi-implicit semi-Lagrangian shallow water limited-area model in the Cartesian geometry. The mass-conserving shallow-water models in spherical geometry were presented in [40,41], however, they use the regular latitude–longitude grid.

In this paper, we address both challenges and describe a mass-conservative version of the shallow-water model [25] implemented on the reduced latitude–longitude grid. We demonstrate that the mass-conservative shallow-water model on the sphere with such a grid has the same accuracy as its non-conservative counterpart on the regular latitude–longitude grid. We also describe some changes in the discrete formulation of the model that increase its accuracy and ease parallel implementation of the model.

The paper is organized as follows. Section 2 reviews different grids used in atmospheric models and describes the reduced latitude–longitude grid applied in the presented model. Section 3 briefly presents the model formulation and numerical methods used. Locally-conservative semi-Lagrangian transport scheme on the reduced grid is described in Section 4. Section 5 describes the mass-conservative discretization for the shallow-water equations. The results of the advection tests and the standard test set for shallow-water models on the sphere are given in Section 6.

2. Construction of the reduced latitude–longitude grid for a finite-difference model

One of the ways to avoid concentration of grid points near the poles on the regular latitude–longitude grid is the so called reduced latitude–longitude grid, i.e. the grid where the number of points in longitude at each latitude circle is gradually diminished while approaching the pole. Richardson [42] was the first who proposed such a grid. This approach was studied in [43,44]. The review of early attempts to use the reduced grid in global finite-difference models is given in [45]. It was found that the reduced latitude–longitude grid provides less accurate solution than the regular grid combined with the longitude filtering in Fourier space.

The experience of first attempts to use the reduced grid in finite-difference atmospheric models has shown that it is very important to construct such a grid properly. In particular, it was recognized that the so called "cosine" grid, i.e. the grid with the quasi-equal step in longitude (in physical space) at all latitudes leads to the unacceptable errors in the solution [45].

The main motivation to use the reduced grid at that time was the limitation on the time step imposed by the Courant–Friedrichs–Lewy (CFL) condition which becomes too strict near the poles. This problem was solved since the introduction of the semi-implicit time stepping by Robert et al. [46], and the reduced grids were forgotten for about 20 years.

The interest in reduced grids was revived by the advances in the development of the global atmospheric models based on spectral transform methods [47]. (Note also the paper [48] on the use of reduced grid for pure advection problem.) It was found again that the reduced grid having quasi-equal step in longitude leads to significant error near the poles [49]. It was then proposed to select the number of grid-points at each latitudinal circle using the asymptotic properties of associated Legendre polynomials, and since then the reduced grid is applied in some spectral atmospheric models [18,50,20]. Construction of the reduced grid based on Legendre polynomials is however not well suited for a grid-point model.

In [51,52], the block-structured reduced grid is applied while solving shallow-water equations on the sphere. Starting from a certain latitude, the number of grid points along latitude circle is reduced by a factor of 2. Such a reduction can be repeated at some other latitude. It turned out that only moderate reduction of grid points is possible without significant error growth near the poles. In [52], this error growth (mainly in velocity components) was attributed to high curvature of curvilinear coordinates near the poles. The way to overcome this difficulty was proposed in [56] – the reduced grid should have more points in longitude at near-pole latitude rows, however, this would be impossible for a reduced grid setup used in [51,52]. Fadeev [53] proposed to construct the reduced grid for a grid-point semi-Lagrangian model considering the accuracy of 2D interpolation on the sphere for an arbitrary position of the interpolation point. This algorithm can be briefly summarized as follows.