Journal of Computational Physics 229 (2010) 8564–8577

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

A stabilized MLPG method for steady state incompressible fluid flow simulation

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ARTICLE INFO

Article history:
Received 8 January 2010
Received in revised form 28 July 2010
Accepted 2 August 2010
Available online 5 August 2010

Keywords:
MLPG
MLS
SUPG
Mixed formulation
Incompressible fluid flow

ABSTRACT

In this paper, the meshless local Petrov–Galerkin (MLPG) method is extended to solve the incompressible fluid flow problems. The streamline upwind Petrov–Galerkin (SUPG) method is applied to overcome oscillations in convection-dominated problems, and the pressure-stabilizing Petrov–Galerkin (PSPG) method is applied to satisfy the so-called Babuška–Brezzi condition. The same stabilization parameter $\tau(\tau_{SUPG} = \tau_{PSPG})$ is used in the present method. The circle domain of support, linear basis, and fourth-order spline weight function are applied to compute the shape function, and Bubnov–Galerkin method is applied to discretize the PDEs. The lid-driven cavity flow, backward facing step flow and natural convection in the square cavity are applied to validate the accuracy and feasibility of the present method. The results show that the stability of the present method is very good and convergent solutions can be obtained at high Reynolds number. The results of the present method are in good agreement with the classical results. It also seems that the present method (which is a truly meshless) is very promising in dealing with the convection- dominated problems.

1. Introduction

It is well known that the mesh-based methods, such as the finite volume method and the finite element method, have been succeeded in dealing with convection-dominated flow problems (see Tao [1]; Zienkiewicz and Taylor [2]). In these methods some stable schemes have been developed to discretize the convection term, so these methods can be applied to solve incompressible flow problems in a wide range of fluid velocity. But these methods meet severe difficulties when they are used to solve some special problems, such as large deformation, shear-band formation, moving boundary, etc., and at the same time, mesh generation is a far more time-consuming and burdensome task than the assembly and solution of the resulting algebraic equations. Owing to these reasons, meshless methods have received much attention in recent decade as a new tool to overcome the above difficulties. A number of meshless methods have been developed by different authors. These include smooth particle hydrodynamics (SPH) [3,4], diffuse element method (DEM) [5], element-free Galerkin (EFG) [6], reproducing kernel particle method (RKPM) [7], finite point method (FPM) [8], partition of unity method (PU) [9], boundary node method (BNM) [10], local boundary integral equation (LBIE) [11], meshless local Petrov–Galerkin method (MLPG) [12], meshless regular local boundary integral equation (MRLBIE) [13], finite cloud method (FCM) [14], point interpolation method (PIM) [15], least-squares collocation meshless method (LSCM) [16], etc. It should be noted that most of these methods, such as DEM, EFG, RKPM, PUM, PIM etc., are global weak form method and not really meshless method, since

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0021-9991/$ - see front matter
doi:10.1016/j.jcp.2010.08.001

they need a background mesh for the numerical integration. SPH, FPM, MLPG, LSCM are all truly meshless methods. The SPH, FPM and LSCM are strong form method with non-element interpolation scheme and require no integration. But these methods are based on point collocation, and the solutions are very sensitive to the selection of the collocation points. The MLPG method is based on a local weak form and easy to deal with different kind of boundary conditions. Remarkable successes of the MLPG method in computational mechanics have been reported in recent years [17–23]. Apart from its successful applications in solid mechanics, the MLPG method has also been extended to simulate heat transfer and fluid flow problems. The first article applying MLPG method to heat conduction in an anisotropic medium was by Sladek et al. [24]; In their work, the Heaviside function was used as the test function; they also extended the MLPG method to 3D heat conduction in an anisotropic medium [25]; In the recent year, they applied MLPG method to solve thermal analysis of Reinsser–mindlin plates and shallow shells [26,27]. Wu et al. [28,29] applied the MLPG method to solve heat conduction problems of irregular domain encountered in engineering. They compared their results with these of the finite volume method (FVM) obtained by the commercial CFD package FLUENT 6.3, and their results demonstrated that the computational precision of MLPG was much better than that of FVM. The MLPG method was first applied to compute convection–diffusion and incompressible flow problems by Lin and Atluri [30,31]. In their work, two kinds of upwind schemes were constructed to overcome oscillations produced by convection term. They applied upwind schemes to solve the incompressible flow problem based on the primitive variable formulation and added the perturbation term to continuity equation to satisfy the Babuška–Brezzi condition. But when these schemes were applied to compute the high Reynolds number problems, the parameter of perturbation term was difficult to determine and it also suffered from the convergent difficulty. Ma [32] applied MLPG to solve two-dimensional nonlinear water wave problems. Wu et al. [33] applied MLPG to solve incompressible flow problems with vorticity-stream function method without addressing the stability problem. Arefmanesh et al. [34] applied MLPG method to compute non-isothermal fluid flow problem with vorticity-stream function method and unity was applied as weighting function. Mohanmmadi [35] constructed a new upwind scheme to compute incompressible flow problems with vorticity-stream function method; in his work, the Heaviside step function and quadratic spline were used as the test functions, and radial basis function (RBF) interpolation was employed on shape function and its derivatives construction. His results showed that his method could give good accuracy. The approaches based on the vorticity-stream function method can satisfy the incompressible mass conservation condition automatically, but it can not be directly extended to solve 3D problem. So the primitive variable method is applied in this paper.

In the present paper, the meshless local Petrov–Galerkin method with MLS interpolation scheme is applied to solve three benchmark problems of lid-driven cavity flow, backward facing flow and natural convection in a square cavity. The streamline upwind Petrov–Galerkin scheme is applied to overcome oscillations produced by convection term and the perturbation term is added to overcome spurious pressure solution.

2. Streamline upwind Petrov–Galerkin (SUPG) method

It is well known that some upwind schemes have been developed to overcome oscillatory solutions caused by convection term.

The pioneering work on upwind scheme in the MLPG method was suggested by Lin and Atluri. In their scheme, the subdomain of test function shifted opposite to the streamline direction, as shown in Figs. 1 and 2. In the Fig. 2, the shifting distance of local sub-domain specifies as γr_i, r_i is the size of support domain of weight function, $\mathbf{n}_i$ is the unit vector of streamline direction at $\mathbf{x}_i$. Their results showed that this method could not obtain convergent solutions for lid-driven cavity flow problem at the high Reynolds number.

SUPG method was proposed by Brooks and Hughes, which had been applied widely in finite element method for fluid mechanics. The theory of SUPG method was set forth in [2,36].

In the SUPG method, the test function is modified by the streamline upwind method, which is defined as following:

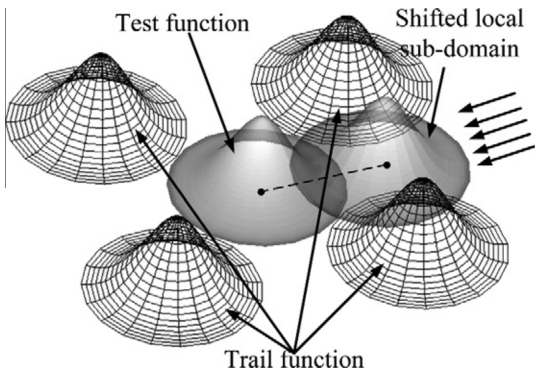

Fig. 1. Upwind scheme (US).