Journal of Computational Physics 269 (2014) 403–422

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Complex-plane generalization of scalar Levin transforms: A robust, rapidly convergent method to compute potentials and fields in multi-layered media

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ARTICLE INFO

Article history:
Received 5 November 2013
Accepted 20 March 2014
Available online 31 March 2014

Keywords:
Sommerfeld integral
Fourier integral
Anisotropic media
Integral convergence acceleration
Method of weighted averages
Multi-layered environment

ABSTRACT

We propose the complex-plane generalization of a powerful algebraic sequence acceleration algorithm, the method of weighted averages (MWA), to guarantee *exponential-cum-algebraic* convergence of Fourier and Fourier–Hankel (F–H) integral transforms. This "complex-plane" MWA, effected via a linear-path detour in the complex plane, results in rapid, absolute convergence of field and potential solutions in multi-layered environments *regardless* of the source-observer geometry and anisotropy/loss of the media present. In this work, we first introduce a new integration path used to evaluate the field contribution arising from the radiation spectra. Subsequently, we (1) exhibit the foundational relations behind the complex-plane extension to a general Levin-type sequence convergence accelerator, (2) specialize this analysis to one member of the Levin transform family (the MWA), (3) address and circumvent restrictions, arising for two-dimensional integrals associated with wave dynamics problems, through minimal complex-plane detour restrictions and a novel partition of the integration domain, (4) develop and compare two formulations based on standard/real-axis MWA variants, and (5) present validation results and convergence characteristics for one of these two formulations.

1. Introduction

In many application areas concerning time-harmonic electromagnetic (EM) fields, one encounters environments containing media of varying and arbitrary anisotropy[1] whose inhomogeneity can be approximated as multi-layered in nature. Examples include geophysical prospection [1–7], plasma physics [8], antenna design [9,10], optical field control [11], microwave remote sensing [12], ground-penetrating radar [13,14], and microwave circuits [15], among others. Such applications regularly encounter integrals of the form[2]

$$f(\mathbf{r}) \sim \iint\limits_{-\infty}^{+\infty} \tilde{f}(k_x, k_y) \mathrm{e}^{ik_x(x-x')+ik_y(y-y')+i\tilde{k}_z(z-z')} \, \mathrm{d}k_x \, \mathrm{d}k_y \tag{1.1}$$

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[1] We assume each medium's anisotropy manifests in *diagonalizable* constitutive material tensors to ensure completeness of the plane wave basis. Since all naturally-occurring media possess diagonalizable material tensors, in practical applications this assumption is always true.

[2] Appendix A summarizes the notation, terminology, and conventions used here.

http://dx.doi.org/10.1016/j.jcp.2014.03.040
0021-9991/

and/or

$$f(\mathbf{r}) \sim \int_{-\infty}^{+\infty} \tilde{f}(k_\rho) H_n^{(1)}\big(k_\rho|\rho-\rho'|\big) e^{i\tilde{k}_z(z-z')}\, dk_\rho \tag{1.2}$$

which express space–domain field/potential functions as Fourier and Fourier–Hankel (F–H) integral transforms (resp.).

In many practical applications, these integrals must often be rapidly evaluated for a wide range of longitudinal and transverse source-observer separation geometries $\mathbf{r}-\mathbf{r}' \neq \mathbf{0}$ (e.g., for potential or field profile reconstruction). However, when using standard integration paths that run on or close to the real axis such as (1) the classic Sommerfeld integration path (SIP) [16] and (2) paths detouring around the branch points, branch cuts, and poles followed by real-axis integration [17–19], the convergence rate of these integrals is strongly dependent upon the transverse $(\mathbf{r}_t-\mathbf{r}'_t)$ and longitudinal $(z-z')$ separations. $\mathbf{r}_t-\mathbf{r}'_t$ determines the rapidity of the integrand's oscillation due to the transverse phase Fourier or Hankel kernel in (1.1) or (1.2) (resp.), with rising $|\rho-\rho'|$ leading to an integrand that traditionally requires increasingly finer sampling to limit spatial aliasing and thus leads to undesirably long computation times. Furthermore, the longitudinal separation $z-z'$ governs the rate at which the evanescent spectrum's field contribution decays with increasing transverse wave number magnitudes,[3] with rising $|z-z'|$ effecting more rapid decay (and hence faster convergence) [20]. On the other hand, as $|z-z'| \to 0$ the convergence rate lessens, with the limiting case $z-z'=0$ yielding integrals of the form

$$f(\mathbf{r}) \sim \iint_{-\infty}^{+\infty} \tilde{f}(k_x,k_y) e^{ik_x(x-x')+ik_y(y-y')}\, dk_x\, dk_y \tag{1.3}$$

and

$$f(\mathbf{r}) \sim \int_{-\infty}^{+\infty} \tilde{f}(k_\rho) H_n^{(1)}\big(k_\rho|\rho-\rho'|\big)\, dk_\rho \tag{1.4}$$

that lead to divergent results when numerically evaluated, using these standard paths, without convergence acceleration.

See Fig. 1 for typical application scenarios wherein these standard paths either succeed or fail to deliver accurate field results. Observing Fig. 1, one immediately realizes that devising an evaluation method for these integrals exhibiting *robustness* with respect to all ranges of $\mathbf{r}-\mathbf{r}' \neq \mathbf{0}$ and medium classes (e.g., isotropic, uniaxial, biaxial) is highly desirable. This robustness criterion inherently excludes fundamentally approximate methods such as image and asymptotic methods due to their geometry-specific applicability and lack of rigorous error control [16,17,21–23]. As a result, to reliably ensure accurate field results for arbitrary environmental medium composition and source-observer geometry combinations, we choose a direct numerical integration method.

In this vein, one option involves pairing standard integration methods with (real-axis path based) algebraic convergence acceleration techniques such as the standard MWA which, based on published numerical results, successfully imparts algebraic convergence acceleration even when $|z-z'|=0$ [18,20]. However, it is desirable to (1) guarantee absolute, exponential convergence in the classical (i.e., Riemann) sense for *any* $\mathbf{r}-\mathbf{r}' \neq \mathbf{0}$ separation geometry (in contrast to only guaranteeing algebraic convergence in the Abel sense when $|z-z'|=0$ [20]) and (2) endow error control to the evanescent-zone field contribution associated with the tail integral, whose relative importance (compared to the radiation-zone contribution) to the field solution grows as $|\mathbf{r}-\mathbf{r}'|$ decreases, to ensure that both the radiation-zone and evanescent-zone contributions are accurately evaluated.[4] To this end, we propose a novel numerical integration method, representing a complex-plane generalization of a specific member of the "scalar Levin-type sequence transform" (SLST) family [24] (i.e., the MWA), that:

1. bends the "extrapolation region"/tail [19] integration path sections to guarantee absolute, exponential convergence of integrals like (1.1)–(1.4),
2. imparts added, *robust* algebraic convergence acceleration to the tail integrals, which compounds with the exponential convergence acceleration to effect absolute, *exponential-cum-algebraic* convergence, via use of a linear path bend combined with our novel, complex-plane generalization of the MWA [18,20],
3. adjusts the detour bend angles to account for the presence of branch points, branch cuts, and poles (summarily referred to here as "critical points"), and
4. addresses the added challenges associated with evaluating *two-dimensional* integral transforms arising as solutions to the wave equation in planar-stratified environments lacking azimuthal symmetry.

[3] I.e., $|k_x|$ and $|k_y|$ for Fourier double-integrals, or $|k_\rho|$ for F–H integrals.

[4] One cannot rely upon a-posteriori error checking, as was done in [18,20], for general environment and source-observer scenarios.