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Classical and semirelativistic magnetohydrodynamics with anisotropic ion pressure

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ABSTRACT

We study the magnetohydrodynamics (MHD) equations with anisotropic ion pressure and isotropic electron pressure under both the classical and semirelativistic approximations in order to develop a numerical model. The dispersion relation as well as the characteristic wave speeds are derived. In addition to the exact wave speed solutions, we also provide efficient approximate formulas for the semirelativistic magnetosonic speeds. The equations are discretized with the Rusanov and Harten-Lax-van Leer numerical schemes and implemented into the BATS-R-US MHD code. We perform a set of verification tests.

1. Introduction

To describe real magnetized plasma, magnetohydrodynamics (MHD) has been widely used in many applications by both the modeling and theoretical communities. There are various types of MHD approximations based on different assumptions and simplifications, which can capture the physical processes of interest. Such processes include, but are not limited to, collisions between particles, Hall current, electric current dissipation, heat conduction, as well as the pressure anisotropy. Pressure anisotropy arises naturally in a low density magnetized plasma, where the gyration and the field-aligned motion of the particles are not coupled by collisions. The magnetic field provides the preferred orientation, while particle collisions tend to drive the plasma isotropic by evenly distributing the parallel and perpendicular momenta with respect to the magnetic field. Without enough collisions, the parallel and perpendicular pressures can be different, however the difference is bounded by instabilities including the firehose, mirror and proton–cyclotron instabilities [1–3]. Space plasmas, our primary interest, are basically collisionless, which means that the pressure anisotropy could play an important role.

MHD with anisotropic pressure was first investigated by Chew et al. [4]. They started from the Boltzmann equation and obtained the Chew–Goldberger–Low (CGL) approximation, also known as the double-adiabatic model, which is valid for single-fluid collisionless plasma with strong magnetic field and neglects the pressure transport along magnetic field lines. Later on Hau and Sonnerup [5] and Hau et al. [6]proposed the double-polytropic model as a more generalized description, which recovers the CGL model as a limiting case. We derive our transport equations by taking the moments of the generalized kinetic equation presented by Gombosi and Rasmussen [7]. We include the electron pressure as well, which is assumed to be isotropic. This assumption is valid in most space plasma applications, since electrons respond to perturbations much more rapidly than ions due to their small mass, as a result their momentum distribution remains approximately isotropic.

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As an important extension to the classical (non-relativistic) case, we study the semirelativistic formulation. The semirelativistic approximation assumes that the plasma flow speed and the sound speed are nonrelativistic, while the Alfvén speed is relativistic. This is applicable for the case when the classical Alfvén speed is comparable or even larger than the speed of light, for example in Jupiter's and Saturn's magnetospheres due to strong planetary magnetic fields. For problems with moderate Alfvén speeds, the semirelativistic form of MHD equations is still useful because it can accelerate numerical convergence to steady state solutions by artificially reducing the speed of light, which is known as the "Boris correction" in the space plasma modeling community [8]. For single-fluid ideal MHD, the semirelativistic equation set as well as characteristic waves were presented in [9].

This is the first time that a numerical model is built to solve the semirelativistic MHD equations with anisotropic ion pressure and isotropic electron pressure. As a first step, we derive the dispersion relation and solve for the characteristic wave speeds. The maximum wave propagation speed determines the maximum stable explicit time step according to the Courant–Friedrichs–Lewy (CFL) stability condition. The maximum wave speed is also required for the Rusanov (or local Lax–Friedrichs) scheme [10], while the fastest left and right wave speeds are needed for the Harten–Lax–van Leer (HLL) scheme [11]. The anisotropic MHD equations are implemented into the BATS-R-US MHD code [12,13], which can solve various forms of the MHD equations including Hall, semirelativistic, multi-species, multi-fluid and so on. The pressure anisotropy is the latest capability of the BATS-R-US code.

The paper first presents the MHD equations for both classical and semirelativistic cases with anisotropic ion pressure and isotropic electron pressure. In Section 3 the characteristic waves are explored for the semirelativistic approximation. The classical case and the case without electron pressure are also obtained. Section 4 describes the numerical method. In Section 5, we present verification tests using the BATS-R-US code. Section 6 contains our conclusions and plans for future work.

2. Equations

In the presence of anisotropic ion pressure and isotropic electron pressure, the pressure tensor can be written as [4,14]

$$\mathbf{P} = (p_\perp + p_e)\mathbf{I} + (p_\parallel - p_\perp)\mathbf{bb} \tag{1}$$

where $\mathbf{I}$ is the identity tensor and $\mathbf{b} = \mathbf{B}/|\mathbf{B}|$ is the unit vector along the magnetic field $\mathbf{B}$. We define $B = |\mathbf{B}|$ as the magnitude of the magnetic field for later use. The electron pressure is denoted by p_e, while $p_\parallel$ and $p_\perp$ describe the parallel and perpendicular ion pressure components with respect to the magnetic field. The average ion scalar pressure thus can be expressed as

$$p = \frac{2p_\perp + p_\parallel}{3} \tag{2}$$

which is the trace of the ion pressure tensor divided by 3.

2.1. Non-relativistic equations

We start with the equation set for non-relativistic MHD in the primitive-variable form

$$\frac{\partial \rho}{\partial t} + (\mathbf{u}\cdot\nabla)\rho + \rho(\nabla\cdot\mathbf{u}) = 0 \tag{3}$$

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla(p_\perp + p_e) + \nabla\cdot[(p_\parallel - p_\perp)\mathbf{bb}] + \frac{1}{\mu_0}\mathbf{B}\times(\nabla\times\mathbf{B}) = 0 \tag{4}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla\times[-(\mathbf{u}\times\mathbf{B})] = 0 \tag{5}$$

$$\frac{\partial p_\parallel}{\partial t} + (\mathbf{u}\cdot\nabla)p_\parallel + p_\parallel(\nabla\cdot\mathbf{u}) + 2p_\parallel\mathbf{b}\cdot(\mathbf{b}\cdot\nabla)\mathbf{u} = 0 \tag{6}$$

$$\frac{\partial p_\perp}{\partial t} + (\mathbf{u}\cdot\nabla)p_\perp + 2p_\perp(\nabla\cdot\mathbf{u}) - p_\perp\mathbf{b}\cdot(\mathbf{b}\cdot\nabla)\mathbf{u} = 0 \tag{7}$$

$$\frac{\partial p_e}{\partial t} + (\mathbf{u}\cdot\nabla)p_e + \frac{5}{3}p_e(\nabla\cdot\mathbf{u}) = 0 \tag{8}$$

where ρ and $\mathbf{u}$ represent the density and velocity, μ_0 is the permeability of vacuum, and the polytropic index is taken to be 5/3. Note that we assume that the ion and electron velocities are equal, thus we do not consider Hall MHD for this study. Also, the collision terms which describe the interactions between ions and electrons as well as wave scatterings are all neglected. Therefore, we are dealing with an 'ideal' three-temperature MHD approximation, i.e., considering the ion parallel pressure, ion perpendicular pressure and electron pressure seperately.

Compared to the isotropic MHD equations, the continuity Eq. (3) and the induction Eq. (5) remain the same. The momentum Eq. (4) contains the pressure tensor (1) instead of the scalar pressure in the isotropic case. The ion pressure components have their individual evolution Eqs. (6) and (7). In the absence of collision terms, the ratio between the two pressure components might achieve unrealistic values. When implementing the equations into BATS-R-US, we add a relaxation term to