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A new volume-of-fluid method with a constructed distance function on general structured grids

Zhaoyuan Wang, Jianming Yang *, Frederick Stern

IIHR – Hydroscience and Engineering, University of Iowa, Iowa City, IA 52242, USA

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ABSTRACT

A second-order volume-of-fluid method (VOF) is presented for interface tracking and sharp interface treatment on general structured grids. Central to the new method is a second-order distance function construction scheme on a general structured grid based on the reconstructed interface. A novel technique is developed for evaluating the interface normal vector using the distance function. With the normal vector, the interface is reconstructed from the volume fraction function via a piecewise linear interface calculation (PLIC) scheme on the computational domain. Several numerical tests are conducted to demonstrate the accuracy and efficiency of the present method. In general, the new VOF method is more efficient than both the high-order level set and the coupled level set and volume-of-fluid (CLSVOF) methods. The results from the new method are better than those from the benchmark VOF method, particularly in the under-resolved regions, and are comparable to those from the CLSVOF method. Breaking waves over a submerged bump and around a wedge-shaped bow are simulated to demonstrate the application of the new method and sharp interface treatment in a two-phase flow solver on curvilinear grids. The computational results are in good agreement with the available experimental measurements.

1. Introduction

The volume-of-fluid (VOF) method has been widely used in simulating two-phase interfacial and free surface flows since it was introduced by Hirt and Nichols in 1981 [8,25]. In the VOF method, the interface is captured by a scalar function, F, which denotes the volume fraction occupied by one fluid, usually the liquid phase, in a computational cell. As an Eulerian-based method on a fixed grid, the VOF method is well suited for flows with large interface distortions and geometrical changes. The VOF function is not continuous across the interface. In order to advect the VOF function, the interface needs to be reconstructed using a geometric technique. The VOF method has an excellent mass conservation property but it lacks accuracy for the direct calculations of normal and curvature due to the discontinuous spatial derivative of the VOF function near the interface. Moreover, the implementation of the VOF method is not straightforward since a complicated geometric procedure is needed for the interface reconstruction. The level set (LS) method pioneered by Osher and Sethian [20] is also an Eulerian-based method and popularly employed for capturing complicated evolving interfaces in many different applications [32,27,21]. In the LS method, the interface is described by the LS function which is defined as a signed distance function. The normal and curvature can be easily and accurately calculated from the continuous and smooth distance function. In contrast to the VOF method, the LS method is easy to implement in both two- and three-dimensional (2D and 3D) coordinates. The precise sub-cell location required in the sharp interface method [11] can be easily defined. The major issue with the LS

* Corresponding author. Tel.: +1 319 335 5749; fax: +1 319 335 5238.
E-mail address: jianming-yang@uiowa.edu (J. Yang).

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method is its susceptibility to the numerical dissipation, which results in a poor mass conservation property. When the level set function is advected, the sharp edges are usually smoothed out and hence the mass loss occurs, which is especially worse when a coarse grid is used. This problem has been addressed in numerous studies and some attempts have been made to remedy it by using higher order schemes [22], adaptive mesh refinement techniques to increase the local grid resolution [30,33], and the hybrid particle level set (PLS) method [6,37]. However, the inherent mass conservation problem still exists.

A coupled level set and volume-of-fluid method (CLSVOF) has been explored in [3,34,28,29,18,36]. The CLSVOF method takes advantage of both the LS and VOF methods where mass conservation is well preserved and the geometric properties, such as normal and curvature, can be easily estimated from the LS function. However, the implementation of the CLSVOF method is not easy. This is because the VOF method alone is already difficult, the additional complexity to the CLSVOF method is the LS re-distance procedure. Implementations of the CLSVOF method are different in the interface reconstruction, VOF advection, and LS re-distance procedures. In the study by Wang et al. [36], the piecewise linear interface construction (PLIC) scheme presented by Gueyffier et al. [7] was employed for the interface reconstruction. A second-order Runge–Kutta method was used to advect the interface. An efficient distance function construction algorithm was developed, which significantly simplifies the complicated geometric procedure by finding the closest point on the reconstructed interface directly regardless of the interface configuration in each computational cell.

All the methods mentioned above were developed based on structured Cartesian grids. However, for flows with complex geometries, such as ship flows, a general structured grid is often needed. In the study by Kothe et al. [14], the VOF method was implemented in the general curvilinear grid where the interface was reconstructed on the physical domain. An analytical solution for the 3D VOF method on a general grid was presented by López and Hernandez [16]. These two methods are based on un-split advection schemes and their implementation is not easy. In [9,19], the VOF method was extended to the curvilinear co-ordinate system by reconstructing the interface on the computational domain instead of the physical domain, where the interface reconstruction schemes devised on the Cartesian grid can be used directly. This significantly simplifies the geometric procedures on the physical domain, reduces the computational cost, and facilitates the split advection operations. Yang et al. [38] and Lv et al. [17] extended the CLSVOF method on 2D triangular and 3D tetrahedral grids, respectively. However, the implementation of the CLSVOF on a general structured grid is not available in the literature. Although the LS method can be easily implemented on a general structured grid and the VOF equation can be solved by reconstructing the interface on the computational domain, the remaining difficulty of the CLSVOF is the distance function construction that involves a complex geometric procedure. It should be noted that the distance function on the physical domain is needed for the sharp interface treatment. The distance function on the computational domain can be obtained following the schemes developed for a Cartesian grid [36], however, it is not a distance function on the physical domain at all. Moreover, it is also expensive to solve the LS equations on a general structured grid if a high order scheme, e.g., HJ-WENO scheme [10], is used.

In the present study, a second-order accurate distance function construction scheme is developed on a general structured grid in the context of the VOF method. The distance function is necessary for estimating the geometrical properties (normal, curvature) in the sharp interface treatment on a general structured grid. The distance function is obtained directly from the reconstructed interface without solving the LS advection and reinitialization equations. The interface is reconstructed using the VOF function and the normal vector calculated using the constructed distance function via a PLIC scheme on the computational domain. A novel treatment is devised for evaluating the normal vector using the distance function. It is more accurate and robuster than the standard procedure, especially, in the under-resolved regions. Several numerical tests are conducted in order to verify the accuracy, efficiency, and capability of the present VOF method. As will be demonstrated in the numerical tests, the new VOF method is more efficient than both the CLSVOF method and the pure LS method in terms of CPU time. The numerical results from the new VOF method are better than the benchmark VOF method particularly for the under-resolved regions, and are comparable to those obtained on the Cartesian grid using the CLSVOF method. Finally, the new VOF method is applied to simulate breaking waves over a submerged bump and around a wedge-shaped bow as two application examples.

2. Numerical methods

2.1. Volume-of-fluid method

In the VOF method, the interface between liquid and gas is tracked by the VOF function, which is defined as the liquid volume fraction in a computational cell. The value of the VOF function in a cell is defined by

$$F(\mathbf{x},t)=\begin{cases}1, \ \text{in the liquid},\\ 0<F<1, \ \text{at the interface},\\ 0, \ \text{in the gas}.\end{cases} \tag{1}$$

The position of the interface is updated by solving the following VOF advection equation,

$$\frac{\partial F}{\partial t}+\mathbf{u}\cdot\nabla F=0. \tag{2}$$