Journal of Computational Physics 266 (2014) 124–138

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Modeling of laser wakefield acceleration in Lorentz boosted frame using EM-PIC code with spectral solver

Peicheng Yu [a,*], Xinlu Xu [b], Viktor K. Decyk [c], Weiming An [a], Jorge Vieira [d], Frank S. Tsung [c], Ricardo A. Fonseca [d,e], Wei Lu [b], Luis O. Silva [d], Warren B. Mori [a,c]

[a] Department of Electrical Engineering, University of California Los Angeles, Los Angeles, CA 90095, USA
[b] Department of Engineering Physics, Tsinghua University, Beijing 100084, China
[c] Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, CA 90095, USA
[d] Instituto Superior Técnico, Lisbon, Portugal
[e] ISCTE – Instituto Universitário de Lisboa, 1649-026, Lisbon, Portugal

ARTICLE INFO

Article history:
Received 10 October 2013
Received in revised form 13 February 2014
Accepted 17 February 2014
Available online 25 February 2014

Keywords:
Particle-in-cell
Plasma simulation
Laser wakefield accelerator
Boosted frame simulation
Spectral solver
Numerical Cerenkov instability

ABSTRACT

Simulating laser wakefield acceleration (LWFA) in a Lorentz boosted frame in which the plasma drifts towards the laser with v_b can speed up the simulation by factors of $\gamma_b^2 = (1 - v_b^2/c^2)^{-1}$. In these simulations the relativistic drifting plasma inevitably induces a high frequency numerical instability that contaminates the interesting physics. Various approaches have been proposed to mitigate this instability. One approach is to solve Maxwell equations in Fourier space (a spectral solver) as this has been shown to suppress the fastest growing modes of this instability in simple test problems using a simple low pass or "ring" or "shell" like filters in Fourier space. We describe the development of a fully parallelized, multi-dimensional, particle-in-cell code that uses a spectral solver to solve Maxwell's equations and that includes the ability to launch a laser using a moving antenna. This new EM-PIC code is called UPIC-EMMA and it is based on the components of the UCLA PIC framework (UPIC). We show that by using UPIC-EMMA, LWFA simulations in the boosted frames with arbitrary γ_b can be conducted without the presence of the numerical instability. We also compare the results of a few LWFA cases for several values of γ_b, including lab frame simulations using OSIRIS, an EM-PIC code with a finite-difference time domain (FDTD) Maxwell solver. These comparisons include cases in both linear and nonlinear regimes. We also investigate some issues associated with numerical dispersion in lab and boosted frame simulations and between FDTD and spectral solvers.

1. Introduction

Laser wakefield acceleration (LWFA) offers the potential to construct compact accelerators that have numerous potential applications including the building blocks for a next generation linear collider and being the driver for compact light sources. As a result, LWFA has attracted extensive interest since it was originally proposed [1], and the last ten years has seen an explosion of experimental results. Due to the strong nonlinear effects that are present in LWFA, developing predictive theoretical models is challenging [2,3]; therefore numerical simulations are critical. In particular, particle-in-cell (PIC)

* Corresponding author.
E-mail address: tpc02@ucla.edu (P. Yu).

http://dx.doi.org/10.1016/j.jcp.2014.02.016
0021-9991/

simulations play a very important role in LWFA research because the PIC algorithm follows the self-consistent interactions of particles through the electromagnetic fields directly calculated from the full set of Maxwell equations. Using a standard PIC code to study a 10 GeV stage in a nonlinear regime takes approximately 1 million core hours on today's computers and a 100 GeV stage would take 100 million core hours. While computing resources now exist to perform a few of such simulations, it is not possible to carry out parameter scans in full three-dimensions. Therefore, reduced models such as combining the ponderomotive guiding center with full PIC [4] for the wake or with quasi-static PIC [5,6] are used for parameter scans. However, while these models are very useful, they cannot model full pump depletion distances and the quasi-static approach cannot model self-injection. Another reduced model that has been recently proposed is to expand the fields in azimuthal mode numbers and truncate the expansion [7]. This can reduce modeling a 3D problem with low azimuthal asymmetry into a similar computational cost as using a 2D r–z code.

Recently, it was shown that by performing the simulation in an optimal Lorentz boosted frame with velocity v_b, the time and space scales to be resolved in a numerical simulation may be minimized [8–10]. The basic idea is that in the boosted frame the plasma length (the laser propagation distance) is Lorentz contracted while the plasma wake wavelength and laser pulse length are Lorentz expanded. The number of laser cycles is an invariant (assuming there is no reflected wave) so the necessary number of cells needed to resolve the laser is also an invariant while the cell size and hence time step are Lorentz expanded. The increase in time step and decrease in the plasma length lead to savings of factors of $\gamma_b^2 = (1 - v_b^2/c^2)^{-1}$ as compared to a lab frame simulation using the so-called moving window [11]. Using such simulations, it has been shown that using a 1–3 PW laser one could generate a 10 GeV electron beam in a self-guided stage and a 50 GeV beam in a channel guided stage [9]. For these cases the savings can be larger than factors of 10^4. However, in the boosted frame LWFA simulations noise from a numerical instability can be an issue. As discussed in [12–18], the noise results from a numerical Cerenkov instability induced by the plasma drifting with relativistic speeds through the grid. According to the dispersion relation this numerical instability is attributed to the coupling between the wave-particle resonances with EM modes (including aliased modes) in the numerical system. The pattern of the instability in Fourier space can be found at the intersections of the EM dispersion relation of the solver used in the simulation algorithm, and the wave-particle resonances [16–18].

In order to mitigate this instability, it is preferable to use an EM solver that eliminates the numerical instability at the main beam resonance. In this case, the instability occurs only at high $|\vec{k}|$ modes which are far away from the physics of interest. As the EM dispersion curves for most finite-difference time domain (FDTD) solvers inevitably bends down (i.e., supports waves with phase velocities less than the speed of light) at high $|\vec{k}|$, numerical instabilities at the main beam resonance are found in these solvers. However, when using a spectral solver that spatially advances the EM fields in Fourier space, the dispersion curve shows no instability pattern at the main beam resonance. In addition, the pattern at the first space aliasing beam mode is found to indeed be located at high $|\vec{k}|$ values that are far away from the interesting physics. For the spectral solver the numerical Cerenkov instability is located at a predicted pattern in $\vec{k}$ space so it can be conveniently eliminated by applying simple filters directly in $\vec{k}$ space.

In this paper we describe the development of a fully parallelized three-dimensional electromagnetic spectral PIC code called UPIC-EMMA that was rapidly built using components of the UCLA PIC framework (UPIC) [19]. Here we use the word "spectral" to mean the fields are expanded using global basis functions. We used a truncated Fourier series and used FFTs to solve the field equations. In other respects, the code uses similar methods as the FDTD PIC codes, including interpolation of particle information to and from a grid [20]. We demonstrate that through the use of appropriate filters, Lorentz boosted frame simulations of LWFA at the optimum frame velocities can be carried out without limitations from the numerical Cerenkov instability. We show that a simple low pass filter with a hard cutoff at $|\vec{k}|$ works very well. This completely eliminates modes with $|\vec{k}|$ above a selected value. Meanwhile, it is not as easy to use such a filter in $|\vec{k}|$ space using an FDTD solver (and such solvers have instabilities at lower $|\vec{k}|$).

As discussed in Refs. [17,18], when using the FDTD code to simulate relativistic plasma drift, an optimized time step has to be chosen to minimize the instability growth rate. While the instability growth rate is minimized, this time step does not lead to complete elimination of the instability and it can lead to further errors in numerical dispersion. Additional smoothing and filtering can help as well, but unlike when using a spectral code the instability cannot be essentially eliminated. For the spectral code, the only errors in numerical dispersion arise from the use of finite time step. Because, it is not necessary to use an optimum time step (nor does one exist), one can minimize the errors in numerical dispersion for the EM waves by choosing smaller time steps if needed. One disadvantage with the spectral code is that it is not easy to use a moving window, however for the optimum γ_b no moving window is needed. We note that the use of a pseudo-spectral-analytical-time-domain (PSATD) algorithm, and its corresponding characters of numerical Cerenkov instability, has recently been discussed and analyzed [22,25,26]. This can easily be included into UPIC-EMMA if the algorithm is shown to have advantages.

We have benchmarked UPIC-EMMA by comparing the 2D and 3D simulation results of LWFA in Lorentz boosted frames with the corresponding OSIRIS [27] lab frame simulations. Good agreement is found between the OSIRIS lab frame simulations, and UPIC-EMMA boosted frame simulations, in both linear, and nonlinear regimes. We also compare UPIC-EMMA simulations for different values of γ_b and excellent agreements are found.

The remainder of this paper is organized as follows. In Section 2 we discuss the numerical instability induced by relativistic drift. In Section 3, we describe the development of UPIC-EMMA, and how using the algorithms in UPIC-EMMA can eliminate the instability induced by relativistic plasma drift. In Section 4, we discuss details of LWFA Lorentz boosted frame