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A high resolution spatially adaptive vortex method for separating flows. Part I: Two-dimensional domains ☆

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ABSTRACT

A grid-free high-resolution spatially-adaptive vortex method for two-dimensional incompressible flow in bounded domains is presented. The computational algorithm is based on operator splitting in which convection and diffusion are handled separately every time step. In the convection step, computational elements are convected with velocities obtained by fast approximations of the Biot–Savart superposition with second-order Runge–Kutta time integration scheme. Diffusion is performed using the smooth redistribution method that employs a Gaussian basis function for vorticity in the interior. Near solid walls, the core functions are modified to conserve circulation. The no-slip boundary condition is enforced by creating of a vortex sheet that is redistributed to neighboring elements using the redistribution method. The proposed method enables accurate and smooth recovery of the vorticity and does not require explicit use of vortex images or occasional re-meshing. Algorithms for reduction in computational cost by accurately removing elements in overcrowded regions and for spatial adaptivity that allows for variable core sizes and variable element spacing are presented. Computations of flow around an impulsively started cylinder for Reynolds number values of 1000, 3000, and 9500 are preformed to investigate various aspects of the proposed method.

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1. Introduction

Vortex methods [10,27,11] are grid-free Lagrangian computational methods originally devised for simulating incompressible fluid flow at large Reynolds number [8]. Presently, vortex methods are capable of handling complex geometries and the associated boundary conditions as well viscous diffusion over a large range of Reynolds number. In these methods, the vorticity field is discretized using vortex elements or "blobs". Operator splitting of the vorticity equation enables convection and diffusion of vorticity to be numerically carried out as separate steps. Convection is simulated by transporting conserved quantities such as circulation along particles' trajectories, where the particles velocities are obtained using a Biot–Savart summation over all the computational elements. Several methods have been developed for solving the diffusion equation including random walk [8], core expansion [24,27], particle strength exchange (PSE) method [16], and redistribution methods [35,26].

Vortex methods have been successfully used to investigate the evolution of vortex sheets [7,36,18], high Reynolds number wakes [38,6], three-dimensional problems [34,21,31,17,1], non-reacting buoyant plumes [19], reacting flows in shear layers [37], co-axial jets [30], and fires [25,19]. High resolution spatially adaptive vortex methods [22,33,3,14] have been

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developed and used for simulation of separating flows and accurate evaluation of lift and drag. Cottet et al. [14] employed variable vortex blobs and associated spatial adaptivity by introducing a mapping between the spatially varying physical domain and a uniform mapped domain. The PSE scheme for diffusion is carried out in the mapped domain. Ploumhans et al. [33] reported results of high resolution simulations of flow over bluff bodies including a cylinder, a square and the 2-D "Apollo" capsule. The variable resolution method of [33] is based on high-order redistribution schemes in the presence of a solid boundary in the context of the PSE scheme. The no-slip boundary condition is enforced by creating a vortex sheet that cancels the slip at the boundary which is then accurately diffused into the flow domain in a conservative manner. Spatial adaptivity is made possible by mapping of the redistribution onto a non-uniform grid which is coarser away from the solid boundary. Barba et al. [3] employed radial basis function (RBF) interpolation techniques to spatial adaption of Lagrangian vortex particles. Core spreading was employed for diffusion with core size control enforced during the adaption process. The adaption process is essentially a mapping to a new set of particles that does not necessarily have to be uniformly spaced. To accurately capture flow separation from a boundary as well as the associated small flow features, near-boundary diffusion should be modeled accurately and the no-slip boundary condition should be properly enforced. Spatial adaptivity is another feature that reduces the computational cost and yet maintains high resolution near the boundary.

Aspects of high resolution vortex methods include (i) fast (multipole) solvers [20,4], (ii) accurate diffusion using the Particle Strength Exchange (PSE) scheme [15,16] and the redistribution method [35,26], (iii) accurate enforcement of the no-slip boundary condition for viscous flow [23,32,28,5], and (iv) spatial adaptivity of elements positions [14,12,11]. Vortex methods primarily differ in the manner they handle the viscous sub-step. The PSE scheme approximates the diffusion operator by an integral which is discretized among neighboring elements. PSE offers spatial adaptivity by remapping elements with variable core size onto uniform blobs [13,14]. Error control in the PSE requires re-meshing, i.e. interpolation from a scattered set of elements to a pre-described mesh. Barba et al. [2,3] discuss the errors incurred by re-meshing and propose a "completely meshless" and spatially adaptive method based on radial basis function (RBF) interpolation.

This paper is organized as follows. First, the vortex method in two dimensions and the no-slip boundary condition implementation are reviewed in Section 2. The smooth redistribution method is then discussed in Section 3 for both flow elements of variable cores as well as boundary sheet elements. Section 4 presents novel algorithms for reduction in the number of elements and for spatial adaptivity using elements with variable cores and variable spacings. The computational algorithm is presented in Section 5. Computing the lift and drag coefficients is discussed in Section 6. In Section 7, various parameters and aspects of method are investigated in terms of the canonical problem of the flow over an impulsively started cylinder and uniform flow over an oscillating cylinder. In this respect, the impact of (i) core-size elements'-spacing overlap, (ii) time step size, and (iii) redistribution length and spatial adaptivity are discussed. Conclusion and future work are finally presented.

2. Two-dimensional vortex method

Vortex methods employ the velocity-vorticity formulation by numerically solving the vorticity transport equation according to the viscous splitting algorithm. In its basic form, this algorithm consists of successive handling of convection and diffusion of vorticity in each time step as follows:

$$\text{convection (inviscid) sub-step} \quad \frac{d\boldsymbol{\omega}}{dt} = 0 \tag{1}$$

$$\text{diffusion (viscous) sub-step} \quad \frac{\partial\boldsymbol{\omega}}{\partial t} = \nu\nabla^2\boldsymbol{\omega} \tag{2}$$

The vorticity field is approximated by the superposition

$$\omega(\mathbf{x},t) = \sum_{i=1}^{N} \Gamma_i(t) f_{\sigma_i}(\mathbf{x},\mathbf{x}_i) \tag{3}$$

where N is the number of vortex elements and f_σ is the basis function of core radius σ. In the convection sub-step, each element is convected according to its velocity which, in the reference frame of a solid body moving with translational velocity $\mathbf{u}_b$, is computed according to the Helmholtz decomposition

$$\mathbf{u} = \mathbf{u}_\omega + \mathbf{u}_\Omega + \mathbf{u}_\infty - \mathbf{u}_b + \mathbf{u}_{ext} \tag{4}$$

where $\mathbf{u}_\omega$ is the vortical velocity component in free space, $\mathbf{u}_\Omega$ is the velocity due to the solid body rotation at an angular speed of Ω, $\mathbf{u}_\infty$ is the free stream velocity, and $\mathbf{u}_{ext}$ is selected such that the no-through flow boundary condition at the solid boundary is satisfied. Depending on the method of enforcing the no-through flow boundary condition, $\mathbf{u}_{ext}$ is the velocity field due to either (i) a vorticity sheet (γ_+) [23,32,28,5] at the solid boundary inside the flow field as depicted in Fig. 1a, or (ii) a potential sheet (q_-) at the solid boundary inside the solid as depicted in Fig. 1b, or (iii) images of vortex elements $(\omega^\star)$ along with a potential sheet $q_{\infty-}$ that cancels the normal component of $\mathbf{u}_\infty - \mathbf{u}_b$ at the solid boundary, as depicted in Fig. 1c [35].

In vortex methods, satisfying the no-slip boundary condition poses a challenge since it is not explicitly a boundary condition for the vorticity. In the context of the viscous splitting algorithm, the no-through flow boundary condition at solid

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