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An interface capturing method with a continuous function: The THINC method with multi-dimensional reconstruction

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ABSTRACT

An interface capturing method with a continuous function is proposed within the framework of the volume-of-fluid (VOF) method. Being different from the traditional VOF methods that require a geometrical reconstruction and identify the interface by a discontinuous Heaviside function, the present method makes use of the hyperbolic tangent function (known as one of the sigmoid type functions) in the tangent of hyperbola interface capturing (THINC) method [F. Xiao, Y. Honma, K. Kono, A simple algebraic interface capturing scheme using hyperbolic tangent function, Int. J. Numer. Methods Fluids 48 (2005) 1023–1040] to retrieve the interface in an algebraic way from the volume-fraction data of multi-component materials. Instead of the 1D reconstruction in the original THINC method, a multi-dimensional hyperbolic tangent function is employed in the present new approach. The present scheme resolves moving interface with geometric faithfulness and compact thickness, and has at least the following advantages: (1) the geometric reconstruction is not required in constructing piecewise approximate functions; (2) besides a piecewise linear interface, curved (quadratic) surface can be easily constructed as well; and (3) the continuous multi-dimensional hyperbolic tangent function allows the direct calculations of derivatives and normal vectors. Numerical benchmark tests including transport of moving interface and incompressible interfacial flows are presented to validate the numerical accuracy for interface capturing and to show the capability for practical problems such as a stationary circular droplet, a drop oscillation, a shear-induced drop deformation and a rising bubble.

1. Introduction

Multi-phase flow analysis is widely required in the fields of science, engineering and medical applications. Many numerical approaches for the multi-component materials have been so far proposed in Lagrangian and/or Eulerian frame.

Among the Lagrangian type methods, the arbitrary Lagrangian Eulerian (ALE) approach [16,5,19,67] has gained a wide popularity because of the accurate treatment of the interface by using a body-fitted grid where the kinematic and dynamic boundary conditions on the interface can be explicitly specified or computed. Further advanced algorithms, such as

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deforming-spatial-domain/space–time (DSD/ST) method, were presented in [51,20] for the flows involving moving and deforming interfaces, where the physical fields are discretized at the space–time nodes to obtain stable and accurate solutions. However, all the methods using the body-fitted grid need a re-meshing process which takes extra time to construct a high quality mesh and is formidably difficult when the interfaces are largely deformed or topologically changed.

The above difficulty can be avoided in fixed-grid approaches in which the moving interface and flow fields are solved on a fixed Eulerian mesh, and thus re-meshing is not needed. However, another issues arising are how to represent and compute the moving interface and the boundary conditions upon it on a fixed mesh. The immersed boundary method [32,33] is one of the most successful methods for these types of problems. Under the framework of the immersed boundary method, the fluid equations are solved in an Eulerian frame, while the interface is tracked in a Lagrangian manner with a set of marker points. The force exerted by the interface on the Eulerian flow field is interpolated with the smoothed (or approximate) delta function. Many improvements and extensions have been proposed over the past decades. For example, a new version of the method was proposed in [23] that achieves a second-order accuracy for representing smooth solutions. The front tracking method [54,12,35,52,53] can be applied to multi-phase flow problems including the surface tension effect with different fluid properties. Moreover, the immersed interface method [24,26] provides a recipe for developing schemes for problems with piecewise smooth solutions, by introducing the modified Taylor expansion with the interfacial jump conditions. In these schemes, the interface can be accurately represented by the Lagrangian particles, which is of particular interests when the interface has structures that are under the resolution of the fixed Eulerian mesh. On the other hand, the particle-based methods do not automatically conserve the volume or mass enclosed by the surface reconstructed from the marker particles, and the numerical treatment of the topological change, such as coalescence or breakup, requires additional physical model [28,29,64].

For representing the interfaces, field variables or indicator functions defined on the Eulerian mesh are also widely used, rather than the Lagrangian particles, in the area of practical simulations involving resolved interfaces. Among the representative field functions used to compute the moving interface are the volume-of-fluid (VOF) function [17], level-set function (signed distance function) [31,45,47], density or color function [59,60,57] and the phase-field function (order parameter) [21,4,8]. For this Eulerian representation of the interface, large efforts have been paid for constructing/maintaining sharp interface. Using field function facilitates treatment of the coalescence or breakup of the free surface. More importantly, numerical conservation can be exactly guaranteed if the field function is updated by a finite volume scheme of flux form, which is naturally implemented in the VOF method.

In the present paper, we focus on the VOF approach where the volume fraction of a specific material (generally called VOF function of the material) is updated by solving the advection equation with a finite volume formulation on a fixed Eulerian mesh. In a conventional VOF method, the interface separating different fluids is piecewisely reconstructed for each mesh (or cell) by straight line segment before calculating the numerical fluxes to update the VOF function. This geometric reconstruction effectively eliminates the numerical diffusion that smears out the compactness of the transition layer of the interface. The accurate recovery to the exact surface geometry from the discretized VOF function, in other words the geometric faithfulness, is closely related to the accuracy of the method. Compared to the reconstruction by the simple line interface calculation (SLIC) method [27], the piecewise linear interface calculation (PLIC) method originally proposed by Youngs [65,66] gives numerical solutions of much superior quality. The successive progress of the PLIC type method can be found in Puckett et al. [36], Rider and Kothe [39], Harvie and Fletcher [15], Aulisa et al. [1] and Pilliod and Puckett [34]. In these PLIC algorithms, an interface is recovered from the volume fraction values by assembling the surface segments encompassing the cut volume, which usually involves the descriptions of planes cutting through the mesh cells. This kind of interface reconstruction is referred to as the geometrical reconstruction in our context.

Especially, Scardovelli and Zaleski [42] derived an elegant formulation to analytically determine the cutting interface from the volume fraction and vice versa, with the given normal direction. Their method substantially reduces the explicit geometrical computation in the traditional PLIC methods, and has been applied to a problem of transporting moving interface in [2].

Further developments followed by taking the quadratic interface into account to improve the numerical accuracy in calculating the geometric properties such as the normal vector and the curvature [38,43] as well as in representing the curved interface so that the interface satisfies the C_1 continuity across the computational mesh [9]. As a different philosophy to increase the accuracy of the curvature estimation, Sussman and Puckett [46] proposed the coupled level set and volume-of-fluid (CLSVOF) method, in which not only the VOF function but also the level-set function are employed to improve the geometric faithfulness of the computed interface.

One of the present authors has proposed another type of VOF method, the tangent of hyperbola for interface capturing (THINC) method [56], to avoid the explicit geometric reconstruction in the conventional VOF methods. In the THINC method, a continuous sigmoid function (1D hyperbolic tangent function) is employed instead of the Heaviside function which allows to represent the interface completely in an algebraic form, thus it enables computation of numerical flux without the geometric information. Based on the continuous sigmoid function, a transition layer across the interface is readily to be controlled as a step-like distribution, and thus can effectively remove the numerical diffusion. The original THINC method [56] employs the 1D reconstruction function and uses directional splitting for multi-dimensional cases. An improvement was proposed by combining the original THINC method and the first-order upwind method. The developed scheme is so-called THINC/WLIC (THINC/weighted linear interface capturing) method [61], in which the weighted average of the numerical fluxes (obtained from the first-order upwind method and the original THINC method) is determined by the respective