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CAD and mesh repair with Radial Basis Functions

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ABSTRACT

In this paper we present a process that includes both model/mesh repair and mesh generation. The repair algorithm is based on an initial mesh that may be either an initial mesh of a dirty CAD model or STL triangulation with many errors such as gaps, overlaps and T-junctions. This initial mesh is then remeshed by computing a discrete parametrization with Radial Basis Functions (RBF's).

We showed in [1] that a discrete parametrization can be computed by solving Partial Differential Equations (PDE's) on an initial correct mesh using finite elements. Paradoxically, the meshless character of the RBF's makes it an attractive numerical method for solving the PDE's for the parametrization in the case where the initial mesh contains errors or holes. In this work, we implement the Orthogonal Gradients method to be described in [2], as a RBF solution method for solving PDE's on arbitrary surfaces.

Different examples show that the presented method is able to deal with errors such as gaps, overlaps, T-junctions and that the resulting meshes are of high quality. Moreover, the presented algorithm can be used as a hole-filling algorithm to repair meshes with undesirable holes. The overall procedure is implemented in the open-source mesh generator Gmsh [3].

1. Introduction

Using CAD data for finite element analysis has become the actual standard in the engineering practice. Yet, geometries that come out of design offices are not free of problems: slivers, cross-overs, surfaces with multiples unnecessary patches, super-small model entities and many other issues that are encountered in the CAD data make the meshing process a nightmare. Those *dirty geometries* are still the cause of time consuming repair processes. The same kind of issues are present when dealing with STL triangulations as the input geometry: the mesh may be noisy, self-intersecting, not watertight, with T-junctions[2] and have undesirable holes. Fig. 1 gives an example of such dirty CAD models or STL triangulations that need to be repaired.

There are two approaches for cleaning dirty geometries: one acts on the CAD model and one acts on the mesh.

The first approach corrects the geometry directly by using point and edge merging algorithms [4–6]. Those approaches provide then specific tools for model correction, controlled primarily by the user [7,8]. Presently, there are also many commercial software modules that claim to be able to perform automatic geometry healing. However, these third party software modules can only rectify common geometry problems and a successful or unsuccessful outcome is possible. Thus there is yet no absolute solution for geometry/mesh healing of CAD models.

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[2] A T-Junction is an intersection of two or more faces in a mesh where the vertex of one face lies on the edge or interior of another face.

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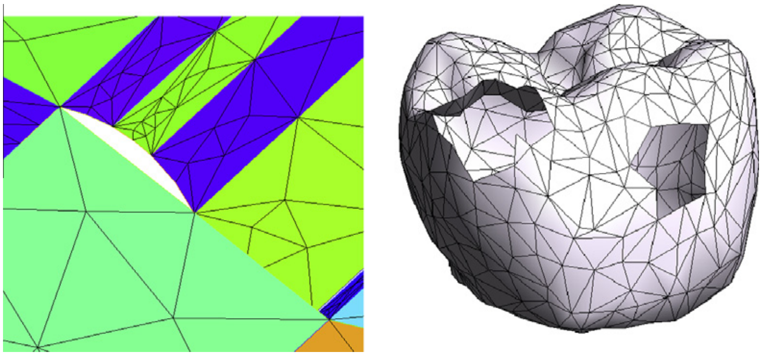

Fig. 1. Two examples for which a CAD and mesh repair algorithm is needed. The figure on the left shows an initial triangulation of a dirty CAD model with topological errors: gaps (holes), overlaps and T-junctions. The right figure displays an STL triangulation of a tooth that contains undesirable holes.

Another approach is that of correcting an initial triangulation of the model through the addition of triangles and different stitching procedures [9–11]. In the same vein, Nooruddin and Turk [12] proposed a method to repair polygonal meshes using volumetric techniques. Unfortunately, not all of those algorithms deal with geometric intersections and/or inconsistent topologies. Also based on an initial triangulation, some authors [13,14] suggested some specific hole-filling algorithms that employ RBF's as an interpolation technique to construct an implicit surface patch and to intersect this implicit surface with the existing triangulation. However, the intersection method to reconstruct the mesh for the hole is a complex process. Moreover, the resulting mesh may contain triangles of low quality when the holes to fill are quite small.

In this paper, an original alternative approach is presented. The repair process includes both model/mesh repair and mesh generation. The remeshing procedure relies on a discrete parametrization. Surface parametrization techniques [15–17] originate mainly from the computer graphics community: they have been used extensively for applying textures onto surfaces [18,19] and have more recently become a very useful and efficient tool for many mesh processing applications [17,20,21]. In the context of remeshing procedures, the initial surface is parametrized onto a planar surface, the surface is meshed using any standard planar mesh generation procedure and the new triangulation is then mapped back to the original surface [22,23]. In recent papers [1,24,25], our group showed that harmonic maps can be computed efficiently by solving partial differential equations (PDE's) on the initial triangulation with finite elements.

However, in the context of CAD and mesh repair, the initial triangulation may contain topological errors such as holes, T-junctions and overlaps. Classical mesh-based numerical techniques for solving PDE's such as finite elements or finite volumes cannot be used with such dirty meshes. The meshless character of the RBF's makes them an attractive alternative for solving those PDE's. Although the RBF method has been used as an interpolation technique since the 1970s, it is only in the 1990s that it was introduced as a technique to solve PDE's [26,27]. Its high accuracy and meshless character have made it the method of choice for problems set on complicated geometries [28]. Often overlooked for having poor stability and high complexity issues, the method has finally gained acknowledgement. A number of studies showed the method's great potential by solving full-scale geophysical applications, and by showing that RBF's could compete with the most trusted numerical techniques [29–33]. Although a good part of the RBF literature deals with surface reconstruction [34,35], or more recently PDE's over spheres [31,30,36], no technique has been developed to solve PDE's on arbitrary surface, until [2], which provides the very first methods, based on RBF's, for solving PDE's on completely arbitrary surfaces. In this work, we implement the RBF's Orthogonal Gradients method of [2] which relies on a level set representation of the initial surface.

The paper is organized as follows. In Section 2, we present the PDE's for solving the parametrization. In Section 3, we show how to solve the PDE's with the RBF Orthogonal Gradients method (OGr) centered on the mesh vertices of the initial triangulation. Section 4 explains how to choose the RBF parameters of the presented Ogr method. Next, we explain in Section 5 how to find the inverse mapping in order to be able to project the new points on the 3D surface. Finally, results are presented to validate the method and to reveal the efficiency and accuracy of our proposed algorithm.

2. Discrete parametrization

The discrete parametrization aims at computing the discrete mapping $\mathbf{u}(\mathbf{x})$ that maps every mesh vertex $\mathbf{x}$ of an initial triangulation of a surface Γ to a point $\mathbf{u}$ of Γ' embedded in $\mathcal{R}^2$:

$$\mathbf{x} = \{x, y, z\} \in \Gamma \subset \mathcal{R}^3 \mapsto \mathbf{u}(\mathbf{x}) = \{u, v\} \in \Gamma' \subset \mathcal{R}^2. \tag{1}$$

We restrict ourselves to the parametrization of surfaces that can be mapped onto a subset of $\mathcal{R}^2$, which means that surfaces we deal with have their genus equal to zero and have at least one boundary. In some recent papers [24,25], efficient techniques have been developed to split objects with complex topologies into simpler ones in the context of surface remeshing.

In this work, we have chosen a harmonic mapping for the parametrization [1,22,23]. Harmonic maps $\mathbf{u}(\mathbf{x})$ can be computed by solving two PDE's that are the Laplace equations on the 3D surface Γ: