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Combining the vortex-in-cell and parallel fast multipole methods for efficient domain decomposition simulations

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Abstract

A new combination of vortex-in-cell and parallel fast multipole methods is presented which allows to efficiently simulate, in parallel, unbounded and half-unbounded vortical flows (flows with one flat wall). In the classical vortex-in-cell (VIC) method, the grid used to solve the Poisson equation is typically taken much larger than the vorticity field region, so as to be able to impose suitable far-field boundary conditions and thus approximate the truly unbounded (or half-unbounded) flow; an alternative is to assume periodicity. This approach leads to a solution that depends on the global grid size and, for large problems, to unmanageable memory and CPU requirements. The idea exploited here is to work on a domain that contains tightly the vorticity field and that can be decomposed in several subdomains on which the exact boundary conditions are obtained using the parallel fast multipole (PFM) method. This amounts to solving a 3-D Poisson equation without requiring any iteration between the subdomains (e.g., no Schwarz iteration required): this is so because the PFM method has a global view of the entire vorticity field and satisfies the far-field condition. The solution obtained by this VIC–PFM combination then corresponds to the simulation of a truly unbounded (or half-unbounded) flow. It requires far less memory and leads to a far better computational efficiency compared to simulations done using either (1) the VIC method alone, or (2) the vortex particle method with PFM solver alone. 3-D unbounded flow validation results are presented: instability, non-linear evolution and decay of a vortex ring (first at a moderate Reynolds number using the sequential version of the method, then at a high Reynolds number using the parallel version); instability and non-linear evolution of a two vortex system in ground effect. Finally, a space-developing simulation of an aircraft vortex wake in ground effect is also presented.

Keywords: Vortex methods; Parallel fast multipole method; Vortex-in-cell method; Particle methods; Lagrangian methods; Incompressible flows; Unsteady flows; Unbounded flows; Wall-bounded flows

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1. Introduction

The method developed here is a combination of the parallel fast multipole (PFM) method and of the vortex-in-cell (VIC) method that allows to efficiently simulate, in 2-D and in 3-D, incompressible unsteady flows that are unbounded or half-unbounded.

Vortex particle methods are based on the vorticity–velocity formulation of the Navier–Stokes equations and on the fact that, for incompressible flows, it is sufficient to follow the evolution of the vorticity field, the velocity field being recovered from the vorticity. See, e.g., Cottet and Koumoutsakos [9], Winckelmans [40] for global reviews. The Lagrangian treatment of the convection term leads to methods with negligible dispersion error. It also eliminates the CFL stability constraint; however, there is still an accuracy constraint that limits the relative rotation of vortex particles ($|\boldsymbol{\omega}|\Delta t$ must remain moderate). Vortex methods also have good energy conservation properties. These two qualities (negligible dissipation and negligible dispersion) make vortex methods suitable candidates for direct numerical simulation (DNS) and/or large-eddy simulation (LES) of complex convection dominated flows.

When using the vorticity–velocity formulation of the incompressible Navier–Stokes equations, one needs to solve a Poisson equation for the streamfunction. Two different approaches are commonly used:

- *Lagrangian vortex method*: the Poisson equation is solved using the unbounded Green's function approach (Biot-Savart). The unbounded domain is thus taken into account implicitly. Thanks to PFM methods [13,6], one is able to obtain, both in 2-D and in 3-D, the streamfunction (and thus the velocity, its gradient, etc.) with an $\mathcal{O}(N \log N)$ computational cost, where N is the number of particles. Those approaches make use of outer multipole expansions (i.e., expansions representing the field outside of a ball). There are also implementations that make use of both inner and outer expansions, allowing to reduce the cost to $\mathcal{O}(N)$. Our parallel 3-D implementation is based on an oct-tree and outer expansions; it also uses active error control based on tight error bounds, allowing to minimize the computational cost required to obtain the field with an error that is uniformly bounded in space at a prescribed level (the error tolerance being an input to the code): see [32,33,27,28]. Another parallel 3-D implementation is that by Krasny et al. [19].
- *Vortex-in-cell method*: in such hybrid Lagrangian–Eulerian methods, the Poisson equation is solved on a grid, see [7,11,12,25]. This is done using fast Poisson solvers. Those also have a computational cost $\mathcal{O}(M \log M)$, where M is the number of grid points, but are considerably faster than the PFM methods. However, boundary conditions are required. Hence, for unbounded (or half-unbounded) flows problems, the grid must be taken much larger than the vorticity field region, so that approximate far-field boundary conditions can be used. In some cases, one assumes periodicity: this too calls for a large grid. In both cases, the obtained solution is dependent on the global grid size, and so are the computational memory and cost requirements. In other implementations, a slightly modified Schwarz alternating algorithm can be used: an iterative method to obtain consistent boundary conditions on each subdomains and that avoids solving a boundary integral problem while still retaining good convergence properties [3,25]. Nevertheless, it results that the VIC approach used "alone" is not really appropriate for unbounded flows, as it does not retain the ability of vortex methods to exactly satisfy the far-field condition.

The present "VIC–PFM" method constitutes a combination of these two approaches. It uses the "smallest" possible domain (i.e., the "smallest" grid): one that tightly contains the vorticity region. In parallel, this domain is further decomposed by splitting it into several subdomains, with one subdomain per processor. The boundary conditions required on the sides of these subgrids are obtained "exactly" by using the PFM method, which has a global view of the entire vorticity field. The Poisson equation is then solved on each subgrid, i.e., locally on each processor, and the combination of all subgrid solutions still provides the solution of a truly unbounded flow. Moreover, from a computational point of view, the efficiency of this method is better than that obtained using a pure PFM method, while benefiting from the powerful parallelization of the fast multipole algorithm. As the grid can be taken compact, it is also better than that obtained using a pure VIC method (when the grid is taken large enough to properly approximate an unbounded domain). The present