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A solution accurate, efficient and stable unsplit staggered mesh scheme for three dimensional magnetohydrodynamics

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ABSTRACT

In this paper, we extend the unsplit staggered mesh scheme (USM) for 2D magnetohydrodynamics (MHD) [D. Lee, A.E. Deane, An unsplit staggered mesh scheme for multidimensional magnetohydrodynamics, J. Comput. Phys. 228 (2009) 952–975] to a full 3D MHD scheme. The scheme is a finite-volume Godunov method consisting of a constrained transport (CT) method and an efficient and accurate single-step, directionally unsplit multidimensional data reconstruction-evolution algorithm, which extends Colella's original 2D corner transport upwind (CTU) method [P. Colella, Multidimensional upwind methods for hyperbolic conservation laws, J. Comput. Phys. 87 (1990) 446–466]. We present two types of data reconstruction-evolution algorithms for 3D: (1) a reduced CTU scheme and (2) a full CTU scheme. The reduced 3D CTU scheme is a variant of a simple 3D extension of Collela's 2D CTU method and is considered as a direct extension from the 2D USM scheme. The full 3D CTU scheme is our primary 3D solver which includes all multidimensional cross-derivative terms for stability. The latter method is logically analogous to the 3D unsplit CTU method by Saltzman [J. Saltzman, An unsplit 3D upwind method for hyperbolic conservation laws, J. Comput. Phys. 115 (1994) 153–168]. The major novelties in our algorithms are twofold. First, we extend the reduced CTU scheme to the full CTU scheme which is able to run with CFL numbers close to unity. Both methods utilize the transverse update technique developed in the 2D USM algorithm to account for transverse fluxes *without* solving intermediate Riemann problems, which in turn gives cost-effective 3D methods by reducing the total number of Riemann solves. The proposed algorithms are simple and efficient especially when including multidimensional MHD terms that maintain in-plane magnetic field dynamics. Second, we introduce a new CT scheme that makes use of proper upwind information in taking averages of electric fields. Our 3D USM schemes can be easily combined with various reconstruction methods (e.g., first-order Godunov, second-order MUSCL-Hancock, third-order PPM and fifth-order WENO), and a wide choice of 1D based Riemann solvers (e.g., local Lax–Friedrichs, HLLE, HLLC, HLLD, and Roe). The 3D USM-MHD solver is available in the University of Chicago Flash Center's official FLASH release.

1. Introduction

Many astrophysical applications involve the study of magnetized flows generating shock waves. Modeling such flows requires numerical solution of the equations of magnetohydrodynamics (MHD) that couple the magnetic field to the gas

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hydrodynamics using Maxwell's equations. A valid computer model needs to capture accurately the nonlinear shock propagation in the magnetized flows without sacrificing computational efficiency and stability.

Obviously, with suitable assumptions about flow symmetries, a simple approach to obtain a computationally efficient model is to solve a reduced system in 1D or 2D instead of 3D. However, a limitation of such reduced systems is that they cannot be used to understand complicated nonlinear physics occurring only in the full 3D situation. Although solving the reduced system can illustrate interesting characteristic features (e.g., the inverse energy cascade in 2D turbulence [37]), it is essential to use 3D simulations term in order to understand the full nonlinear nature of MHD phenomena (e.g., the energy cascade from large scales to small scales in 3D turbulence).

There are two approaches in modeling multidimensional (i.e., 2D and 3D) algorithms for gas hydrodynamics and MHD in terms of spatial integration methods: split and unsplit. The directionally split method has the advantage of extending a 1D algorithm to higher dimensions, simply by conducting directional sweeps along additional dimensions, in which each sweep solves 1D sub-system. Thus, the Courant–Friedrichs–Lewy (CFL) numerical stability constraint of the split schemes in multidimensions is the same as the 1D constraint, which is to say CFL $\leqslant 1.0$. Despite their simplicity and robustness, however, a number of recent studies have revealed numerical problems in the split formulations of multidimensional MHD and gas hydrodynamics (e.g., loss of expected flow symmetries [2,39], failure to preserve in-plane magnetic field evolution [28,38], numerical artifacts due to a failure to compute proper strain rates on a grid scale [3]).

For MHD the use of an unsplit formulation is more critical than for hydrodynamics. This is because the split formulations fail to evolve the normal (in the sweep direction) magnetic field [17,28,29,57]. For 2D MHD, Gardiner and Stone [28] identified the importance of such multidimensional consideration in their unsplit MHD scheme based on the corner transport upwind (CTU) [15] and the constrained transport (CT) [24] methods. Later, the authors proposed a 3D unsplit version of an unsplit MHD scheme in [29], in which the extension of the multidimensional MHD terms from their 2D algorithm to 3D is accomplished at the cost of considerable algorithmic complexity and a reduced stability limit (CFL <0.5) in their 6-solve CTU + CT algorithm. It is known in a CTU-type 3D unsplit formulation that the full CFL stability limit (i.e., CFL number $\leqslant 1.0$) can be recovered by accounting for intermediate Riemann problems fully, requiring 12 Riemann solves per zone per time step [53]. In general, the calculations associated with the Riemann solves are computationally expensive. Gardiner and Stone [29] considered two alternative options, an expensive 12-Riemann solve yielding the full CFL limit and a reduced 6-Riemann solve with a more constraining CFL condition (CFL number <0.5). They found that the two approaches are similar in terms of computational cost and there is no significant difference in performance between them. The 6-solve scheme is chosen to be their primary 3D integrator because of its relatively low complexity in incorporating the multidimensional MHD terms.

The CTU formulation has an advantage in its compact design of one-step temporal update which is well-suited for multidimensional problems. However, it is limited to second-order. There has been much progress in other types of temporal update strategies that are higher than second-order accurate, taking a different path from CTU. Early attempts have utilized a Runge–Kutta (RK) based temporal update formulation coupled with spatially high-order reconstruction schemes in the finite-difference framework [33,55,56,12,58,42,36,5]. Such RK-based high-order schemes have been also developed in the finite-volume framework [7,35,67,20,21] which has superior properties to that of finite-difference for resolving compressible flows on both uniform and AMR grids. The high-order RK temporal update strategies rely on multi-stage updates which add to the computational cost. Therefore it is desirable to retain a CTU-like one-step formulation, while retaining higher than second order accuracy. Recent work has been found to provide such efficiency using a new formulation so-called the arbitrary derivative Riemann problem (ADER), see [59,60,62,22,23,8,11]. For solving multidimensional conservation laws, there has been another line of progress that tries to build genuinely multidimensional Riemann solvers for hydrodynamics [1,25,26,30,65,14]. Recently, a family of two-dimensional HLL-type Riemann solvers, HLLE [9] and HLLC [10], have been introduced and generalized by Balsara for both hydrodynamics and MHD. As shown in his work the multidimensional Riemann solvers are genuinely derived for 2D. A major improvement in MHD flows is that they inherently provide proper amount of numerical dissipation that is necessary to propagate magnetic fields in a stable manner. Alternatively, 1D Riemann solver formulations such as [43,28] need to add extra dissipation for a stable upwinding. The use of multidimensional Riemann solvers is also shown to capture isotropic wave propagations better than the usual 1D approach. Furthermore, both types of solvers have been extended to 3D using a one-step predictor–corrector formulation.

The above mentioned strategies using high-order schemes and genuinely multidimensional Riemann solvers, provide improved solution accuracy and stability over CTU-CT formulations. In this paper, however, we are primarily interested in constructing a scheme that can be built on the 1D Riemann solver framework in line with a CTU-type method. The latter is (arguably) most widely used in many Godunov-type modern codes [15,53,41,43,28,29,38,44,45]. This design also benefits us in extending our 2D USM-MHD algorithm [38] to 3D without any modifications of the Riemann solvers. This paper describes an approach that provides (i) an algorithmic extension from 2D to 3D of the USM scheme of Lee and Deane [38], and (ii) the full CFL stability bound in 3D *without* the expense of 12 Riemann solves per cell per time step, and (iii) a new upwind biased electric fields construction scheme for CT. We show that the present USM scheme achieves a numerically efficient and consistent MHD algorithm in 3D without introducing a greater amount of additional complexity, while maintaining the full CFL stability range.

The paper is organized as follows: Section 2 describes our new 3D unsplit, single-step data reconstruction-evolution USM algorithm which consists of two stages, i.e., normal predictor and transverse corrector. Section 2 is subdivided into several subsections. We begin in Section 2.1 our discussion of the 3D USM scheme by considering the governing equations of MHD and their linearized form. The second-order MUSCL-Hancock approach for calculating the normal predictor is described in