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Adaptive mesh finite-volume calculation of 2D lid-cavity corner vortices

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ABSTRACT

An anisotropic refinement criterion suitable for Finite-Volume methods is presented and Navier–Stokes solutions are reported for the lid-driven cavity flow configuration at $Re = 1000$ with adaptive anisotropic meshes (*h*-refinement). The *a posteriori* error estimation criterion is based on the assessment of the goodness-of-fit of the least squares regression used to perform the variables profile reconstruction and it is capable of detecting both large-scale and small-scale flow phenomena. The criterion allowed to capture in detail the large-scale flow structure and also the sequence of creeping flow small sharp corner's [1] eddies, up to the fourth corner vortex in addition to the primary cavity vortex. The smallest corner vortex detected is $O(10^{-16})$ smaller in velocity magnitude compared with the cavity primary recirculation flow.

1. Introduction

There are a number of adaptive methods in Computational Fluid Mechanics to reduce and control the numerical error, either by modification of the mesh, *h*-refinement, or by changing the order of approximation of the numerical scheme, *p*-refinement. These methods require an appropriate error measure to detect the numerical error. There are several error estimation techniques and most of them have been widely used in the context of Finite-Element methods, see e.g. [2–6].

Despite the recent growth of adaptive grid refinement methods for Finite-Volume (FV) calculations, few investigations have been concerned with the chosen *a posteriori* error estimate, also known as refinement criterion. Usually, the refinement criterion is based on the analysis of the absolute or relative importance of the gradient and the Hessian matrix, or in some specific *ad hoc* heuristic measure of detection of important flow features, see [7]. Both approaches have the inconvenient of requiring some empirical manual tuning that are problem dependent, see [8]. Other methods involve the full integration of the governing equations with a low and high order method, see [9]. If the (normalized) difference between the two solutions is greater than a certain threshold then the corresponding cells are marked for refinement. This method involves the full integration of the governing equations under higher order requirements.

An alternative method consists in the estimation of the Taylor series truncation error (TSTE), but if we use this value directly, the threshold value will be dependent on the magnitude of the variables' values, thus requiring user input and knowledge. If we nondimensionalize it with the local resolved terms, the criterion will be overly sensible to variable values near zero, and it will also remain unbounded. Methods based on the estimate of the error of the variables' second moment have been proposed by Jasak and Gosman [10], but it tends to underperform in fine meshes. Moreover, with the exception of the

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TSTE, none of these methods have the ability to provide directional information, limiting the possibility of anisotropic refinement.

We use an anisotropic refinement criterion to reduce the outlined problems. The method is based on performing regression quality diagnostics on the dependent variables' local profiles, since this criterion directly tests the profile assumed by the FV numerical method by considering this as a problem of model building. The method potentially allows to use a suite of powerful model building and regression diagnostic tools. This method is possible to use simultaneously for several dependent variables and combinations, and most importantly, it has the ability to detect both small-scale and large-scale flow features present in the flow. Moreover, it is limited, normalized, while being easy to implement, inexpensive to compute and easy to calibrate.

This paper focuses on a FV implementation of the proposed anisotropic refinement criterion in an adaptive method for two dimensional incompressible flows. The application example is the standard lid-driven cavity flow for Reynolds number $Re = 1000$, see e.g. [11–16], for which a number of well documented benchmark solutions are available (e.g. [17–19]). Ham et al. [20] have considered the same case for $Re = 400$ for cartesian grid method with anisotropic adaptation driven by an error estimator based on the second derivative of the fluid velocities and mesh size.

The lid-driven cavity flow on the vicinity of the lower corners displays multiple eddies in creeping flow regime. Moffatt [1] has shown that under the assumption of 2D Stokes flow, the sharp corner flows formed by two intersection boundaries, where either the velocity or the tangential stress vanishes on each boundary with a critical angle opening, contain a sequence of vortices descending into the corner.

Corner eddies are very small compared to the cavity scale and tend to have little impact on the nature of the bulk flow. However, their description is relevant to understand the flow topology in increasingly slender cavities and the influence on the limiting cases of the aspect cavity ratio, approaching zero or infinite, see [21]. Other theoretical implications are related with 3D corner flows, see [22,23].

The numerical resolution of corner flows has been pursued and constitutes a challenging task in terms of numerical resolution, efficiency, robustness, adaptive mesh refinement and refinement criteria. One of the first attempts, and maybe the most impressive, was conducted by Gustafson and Leben [24] that computed the Stokes flow solution in an uniform grid in the whole domain and then projected it on finer local grids near the corner. A sequence of domains, in a zoom like fashion, allowed to predict up to 21 corner vortices with the local maximum stream function intensity of 10^{-92}. The lack of global interaction prevents their solution to be a benchmark test case for Navier–Stokes solvers. Several other authors combine the solutions of the Navier–Stokes equations with exponential mesh refinement of the cavity corners flow regions and asymptotic of the flow near corners to predict the series of vortices, see [25].

The main objective of this work is to present an adaptive criterion useful for problems with disparate length scales when adaptive meshing is often indispensable for resolving small flow features. To the Authors' knowledge, this work represents the first implementation of a grid refinement criterion for viscous flows within a FV framework that automatically captured multiple eddies, up to the fifth, in the vicinity of each of the lid-driven cavity bottom sharp corners.

The paper is organized as follows: the next section describes the FV method used to solve the flow in hybrid unstructured grids with cells of arbitrary topology. This is followed by the presentation of the refinement criterion and the application of the lid-driven cavity flow at $Re = 1000$. The paper ends with summary conclusions.

2. Mathematical and numerical model

The isothermal flow of an incompressible fluid is governed by the mass and momentum conservation laws, being expressed by the incompressibility constraint and the Navier–Stokes equations:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) = \nabla \cdot \left(v\nabla \mathbf{u} + v\nabla^T \mathbf{u}\right) - \nabla p, \tag{2}$$

where $\mathbf{u}$ is the velocity vector, v is the kinematic viscosity and p is the pressure density ratio. A fractional-step numerical method is used to numerically solve Eqs. (1) and (2). The method starts by computing from a previously existing velocity field $\mathbf{u}^n$ an initial approximation $\mathbf{u}^*$ of the new velocity field. This approximation satisfies the momentum equations without the pressure gradient contribution:

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} = -\nabla \cdot (\mathbf{u}^n \otimes \mathbf{u}^n) + \nabla \cdot \left(v\nabla \mathbf{u}^n + v\nabla^T \mathbf{u}^n\right). \tag{3}$$

$\mathbf{u}^*$ is then projected onto the space of solenoidal fields, by removing its irrotational component:

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} = -\nabla p^{n+1}. \tag{4}$$

Taking the divergence of Eq. (4) and setting $\nabla \cdot \mathbf{u}^{n+1} = 0$ results in the Poisson equation for the pressure field:

$$\nabla^2 p^{n+1} = \frac{\nabla \cdot \mathbf{u}^*}{\Delta t}. \tag{5}$$