Journal of Computational Physics 231 (2012) 745–758

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Parallel filtering in global gyrokinetic simulations

S. Jolliet [a,*], B.F. McMillan [b], L. Villard [b], T. Vernay [b], P. Angelino [b,1], T.M. Tran [b], S. Brunner [b], A. Bottino [c], Y. Idomura [a]

[a] Japan Atomic Energy Agency, Higashi-Ueno 6-9-3, Taitou, Tokyo 110-0015, Japan
[b] Centre de Recherches en Physique des Plasmas, Association Euratom-Confédération Suisse, Ecole Polytechnique Fédérale de Lausanne, PPB, 1015 Lausanne, Switzerland
[c] Max Planck Institut fuer Plasmaphysik, IPP-EURATOM Association, Garching, Germany

ARTICLE INFO

Article history:
Available online 28 January 2011

Keywords:
Gyrokinetics
ITG
Field solver

ABSTRACT

In this work, a Fourier solver [B.F. McMillan, S. Jolliet, A. Bottino, P. Angelino, T.M. Tran, L. Villard, Comp. Phys. Commun. 181 (2010) 715] is implemented in the global Eulerian gyrokinetic code GT5D [Y. Idomura, H. Urano, N. Aiba, S. Tokuda, Nucl. Fusion 49 (2009) 065029] and in the global Particle-In-Cell code ORB5 [S. Jolliet, A. Bottino, P. Angelino, R. Hatzky, T.M. Tran, B.F. McMillan, O. Sauter, K. Appert, Y. Idomura, L. Villard, Comp. Phys. Commun. 177 (2007) 409] in order to reduce the memory of the matrix associated with the field equation. This scheme is verified with linear and nonlinear simulations of turbulence. It is demonstrated that the straight-field-line angle is the coordinate that optimizes the Fourier solver, that both linear and nonlinear turbulent states are unaffected by the parallel filtering, and that the $k_\parallel$ spectrum is independent of plasma size at fixed normalized poloidal wave number.

1. Introduction

It is now commonly accepted that plasma turbulence is responsible for the anomalous transport observed in Tokamaks [1]. The best numerical tools to study this issue are gyrokinetic codes, which consistently solve the Vlasov–Maxwell system [2]. The gyrokinetic theory assumes that the typical frequency of micro-instabilities is much smaller than the cyclotron frequency, thus reducing the number of dimensions from 6 to 5. However, when deriving gyrokinetic equations (see for e.g. [3]), other small parameters are introduced: this is called the gyrokinetic ordering. In particular, it is assumed that the parallel wavenumber $k_\parallel$ is small ($k_\parallel \rho_s \sim \mathcal{O}(\rho^*)$, $\rho^* = \rho_s/a \sim 10^{-2} - 10^{-3}$, where ρ_s is the ion sound gyroradius and a is the minor radius of the Tokamak) whereas the perpendicular wavenumber can be large ($k_\perp \rho_s \sim \mathcal{O}(1)$). This assumption is based on the theoretical argument that small parallel wavelengths are Landau damped and has been observed experimentally [4]. This strong anisotropy of plasma turbulence is the starting point of the so-called flux-tube codes [5], which solve the turbulence on a field-aligned domain. Field-aligned coordinates allow a huge reduction of computational requirements, but may unfortunately be inconvenient once used in a global code due to the magnetic shear: non-rational field lines do not close on themselves and special care must be employed to ensure the poloidal periodicity of the perturbations. Several techniques exist to avoid this problem. In [6], the shifting-metric procedure is applied to have a locally orthogonal coordinate system at each poloidal plane. Another useful technique is to use quasi-ballooning coordinates [7], where the parallel coordinate is not exactly aligned to give straightforward boundary conditions in the poloidal and toroidal directions. This technique is used

* Corresponding author.
E-mail address: jolliet.sebastien@jaea.go.jp (S. Jolliet).
[1] Now at Institute of Chemical Sciences and Engineering, Ecole Polytechnique Fédérale de Lausanne, Switzerland.

0021-9991/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.jcp.2011.01.029

by several global gyrokinetic codes [8–10]. Quasi-ballooning coordinates result in an improved scaling of CPU time ($\propto(\rho^*)^{-2}$) compared to the $(\rho^*)^{-3}$ scaling with standard unaligned coordinates, due to the low resolution needed in the parallel direction. These coordinates act as a natural filter for unwanted high frequencies and allow larger time steps. However, implementing a field-aligned solver is rather complicated due to non-rectangular grids and the treatment of the magnetic axis is generally avoided. Due to magnetic shear, the grid is distorted which may result in poor resolution for radial derivatives. This can be resolved by using an unstructured grid [34]. Unfortunately, the field equation is in that case solved with an iterative solver whose convergence is not guaranteed and depends on the physical problem. For these reasons, many of the global gyrokinetic codes [23,11–13] still solve the field equation on the poloidal plane with direct solvers. The GT3D code [23] uses a ballooning phase factor to extract analytically the $k_\parallel = 0$ structure at a given magnetic surface: the grid resolution of the field equation can be strongly reduced. However, when solving the field equation on the poloidal plane, the discretized spectrum may contain modes with $k_\parallel\rho_s \gg \rho^*$ that are unphysical. Indeed, Particle-In-Cell (PIC) simulations may be polluted by high parallel components generated by inherent statistical noise [12]. This has been resolved by applying a Fourier filter specifically designed to remove high $k_\parallel\rho_s$ modes on the perturbed density [12]. This scheme has been recently improved by applying the same filtering procedure to the potential [14] and results in massive computational savings in global Particle-In-Cell (PIC) codes: memory is decreased by two orders of magnitude, the number of Fourier modes is decreased by one order of magnitude and the number of particles required for a given accuracy is decreased by the same factor as the number of particle per Fourier modes dictates the noise level [15]. It means that in PIC codes, parallel filtering cannot be dissociated from the number of markers. On the other hand, Eulerian codes are free from such noise, and influences of the filtering can be clearly estimated by comparing filtered and non-filtered simulations at a fixed number of grid points. Therefore, this work presents the implementation of the Fourier filtering technique in the Eulerian code GT5D [11] and in the PIC code ORB5 [12].

The rest of this paper is organized as follows. Section 2 briefly presents both codes, the implementation of the Fourier solver [14], and further focuses on the choice of the poloidal angle. The solver is verified with linear and nonlinear simulations in Section 3. Then, turbulent spectra are studied in Section 4, and conclusions are given in Section 5.

2. Implementation of the Fourier solver

2.1. The GT5D code

The detailed implementation of the GT5D code can be found in Refs. [11,16]. It is briefly summarized for completeness. GT5D is a five-dimensional full-f Vlasov code that solves a gyrokinetic equation [17] in Tokamaks:

$$\frac{\partial \mathcal{J}f}{\partial t} + \nabla\cdot(\mathcal{J}\dot{\mathbf{R}}f) + \frac{\partial}{\partial v_\parallel}(\mathcal{J}\dot{v}_\parallel f) = \mathcal{J}[C(f) + S_{\mathrm{src}}(f) + S_{\mathrm{snk}}(f)] \tag{1}$$

where $f(\mathbf{R}, v_\parallel, \mu, t)$ is the ion guiding-center distribution function, $\mathbf{R}$ is the guiding-center position, $v_\parallel$ is the velocity parallel to the magnetic field, μ is the magnetic moment and $\mathcal{J}$ is the phase-space Jacobian. The nonlinear equations of motion $(\dot{\mathbf{R}},\ \dot{v}_\parallel)$ are obtained from a Hamiltonian approach:

$$H = \frac{1}{2}m_i v_\parallel^2 + \mu B + q_i\langle\phi\rangle_\alpha \tag{2}$$

$$\dot{\mathbf{R}} = v_\parallel\mathbf{b} + \frac{c}{q_i B_\parallel^*}\times\left(q_i\nabla\langle\phi\rangle_\alpha + m_i v_\parallel^2\mathbf{b}\cdot\nabla\mathbf{b} + \mu\nabla B\right) \tag{3}$$

$$\dot{v}_\parallel = -\frac{\mathbf{B}^*}{m_i B_\parallel^*}\cdot(q_i\nabla\langle\phi\rangle_\alpha + \mu\nabla B) \tag{4}$$

where $\mathbf{B} = B\mathbf{b}$ is the magnetic field, $\mathbf{B}^* = \mathbf{B} + Bv_\parallel/\Omega_i\nabla\times\mathbf{b}, B_\parallel^* = \mathbf{b}\cdot\mathbf{B}^*, \Omega_i = q_iB/(m_ic)$ is the cyclotron frequency and $\langle\cdot\rangle_\alpha = 1/(2\pi)\oint\cdot\mathrm{d}\alpha$ is the gyro-averaging operator where α is the gyro-phase angle. The equations of motion are obtained through $\dot{\mathbf{R}}\equiv\{\mathbf{R}, H\}$ and $\dot{v}_\parallel\equiv\{v_\parallel, H\}$, where $\{F,G\}$ is the Poisson bracket operator defined by:

$$\{F,G\}\equiv\frac{\Omega_i}{B}\left(\frac{\partial F}{\partial\alpha}\frac{\partial G}{\partial\mu} - \frac{\partial G}{\partial\alpha}\frac{\partial F}{\partial\mu}\right) + \frac{\mathbf{B}^*}{m_iB_\parallel^*}\cdot\left(\nabla F\frac{\partial G}{\partial v_\parallel} - \nabla G\frac{\partial F}{\partial v_\parallel}\right) - \frac{c}{q_iB_\parallel^*}\mathbf{b}\cdot(\nabla F\times\nabla G) \tag{5}$$

The collision operator $C(f)$ is a linearized, drift-kinetic Fokker–Planck operator [18] $C(f)\equiv C_T(\delta f) + C_F(f)$, where $C_T(\delta f)$ is the test-particle operator and $C_F(f)$ is the field-particle operator. In particular, the field-particle operator is constructed numerically in order to conserve density, parallel momentum and energy up to machine precision [19]. Finite Larmor Radius (FLR) effects are neglected.

The source operator is $S_{\mathrm{src}} = A_{\mathrm{src}}(\mathbf{R})\tau_{\mathrm{src}}^{-1}(f_{M1} - f_{M2})$, where A_{src} is a deposition profile, f_{M1} and f_{M2} are (shifted) Maxwellian distributions and τ_{src} is a time constant. τ_{src} is set by imposing zero particle and momentum input, but a fixed power input P_{in}:

$$0 = \int S_{\mathrm{src}}\mathrm{d}^6Z = \int m_iv_\parallel S_{\mathrm{src}}\mathrm{d}^6Z \tag{6}$$