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Review of implicit methods for the magnetohydrodynamic description of magnetically confined plasmas

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ABSTRACT

Implicit algorithms are essential for predicting the slow growth and saturation of global instabilities in today's magnetically confined fusion plasma experiments. Present day algorithms for obtaining implicit solutions to the magnetohydrodynamic (MHD) equations for highly magnetized plasma have their roots in algorithms used in the 1960s and 1970s. However, today's computers and modern linear and non-linear solver techniques make practical much more comprehensive implicit algorithms than were previously possible. Combining these advanced implicit algorithms with highly accurate spatial representations of the vector fields describing the plasma flow and magnetic fields and with improved methods of calculating anisotropic thermal conduction now makes possible simulations of fusion experiments using realistic values of plasma parameters and actual configuration geometry. This article is a review of these developments.

1. Introduction

This article is a review of the progress made during the last 35 years in developing accurate and efficient implicit algorithms for simulating the global dynamics of strongly magnetized low-β (ratio of plasma to magnetic pressure) plasmas such as exist in modern magnetic fusion experiments; in particular the tokamak configuration [1]. Other, related confinement configurations that these methods are useful for include stellarators, reversed field pinches, spheromaks, and spherical tori. We limit our discussion to algorithms for solving the magnetohydrodynamic (MHD) equations [2], in which the plasma is described as a conducting fluid. Several forms of these equations are summarized in Appendix B. Efforts at extending this work to include intrinsically kinetic effects are presently underway, but this will not be covered in the present review.

The typical geometry of a tokamak experiment is shown in Fig. 1 where a cylindrical (R,φ,Z) coordinate system is used. The equilibrium magnetic field is axisymmetric; independent of the toroidal angle φ. The magnetic field is composed of a toroidal field, which is into the plane of the paper, and a poloidal field, which lies within the plane of the paper. The magnetic field lines interior to the separatrix surface form closed flux surfaces on which the temperature, pressure, and density are nearly constant. These are shown as solid curves in the figure. The ratio of the number of times a magnetic field line goes the long way around the torus to the number of times it goes the short way around on one of these surfaces is called the safety factor, which we denote as q. It typically varies from 1 near the magnetic axis to 3 or 4 near the edge. Exterior to the separatrix surface, the magnetic surfaces are open and the field lines intersect the vacuum vessel or other structures. The plasma on these open surfaces will necessarily be very low pressure, density, and temperature. The vacuum vessel is sometimes modeled as a perfect conductor, but in reality has some electrical resistance, which can be important for some plasma dynamics [3,4]. Exterior to the vacuum vessel, it is normally assumed that a vacuum exists so that the free-space

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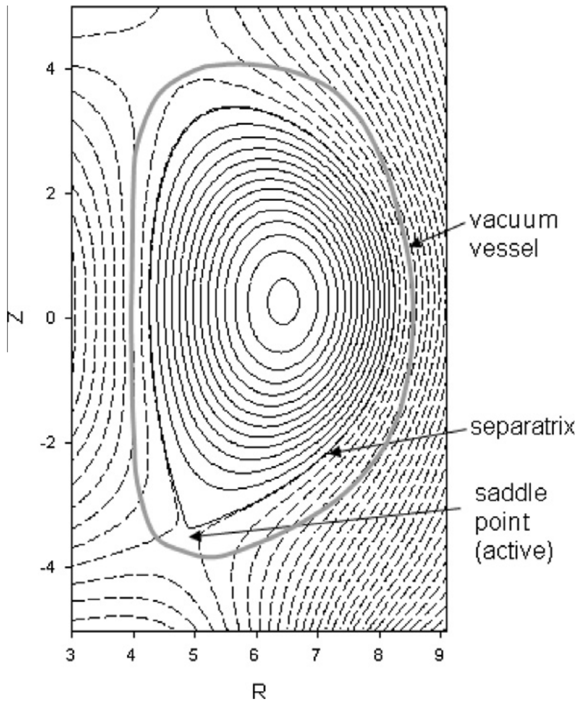

Fig. 1. In a tokamak, the equilibrium plasma is axisymmetric. Magnetic flux surfaces are closed interior to the separatrix and open exterior to it. Plasma is surrounded by a metallic vacuum vessel.

Maxwell equations are satisfied. This is the geometry we are interested in simulating, although much of the algorithm development leading up to simulations in this geometry has occurred using much simpler geometry.

In Section 2 we review the reasons for the multiple timescales that exist when describing the global dynamics of magnetized plasma, and hence the need for an implicit algorithm. Section 3 traces back the origins of modern algorithms for treating the hyperbolic terms (ideal MHD) to similar methods proposed for implicit hydrodynamics in the 1960s. In Section 4 we discuss some of the considerations in choosing a spatial representation, including the choice of variables used in representing the vector fields. We discuss implicit treatment of the anisotropic heat conduction in Section 5, and techniques for dealing with the terms that occur in the two-fluid (2F) description in Section 6. Section 7 contains a short summary and some observations. In Appendix A, we show the relation of the algorithms most widely used for implicit treatment of the hyperbolic terms to the Schur complement of a matrix.

2. The need for an implicit algorithm

Global plasma instabilities are termed either ''ideal'' or ''resistive'' depending on the minimum equation set that is required to describe their onset. Ideal instabilities require only the ideal MHD equations [5], while resistive instabilities require the presence of resistivity, or other forms of dissipation. In general, resistive instabilities occur on significantly slower timescales than do ideal instabilities since the resistivity in high temperature plasma is very low.

There is a wide separation in timescales even within phenomena described by the ideal MHD equations in tokamak geometry. There are three characteristic wave propagation velocities in ideal MHD: that of the slow wave V_S, the Alfvén wave V_A, and the fast wave V_F. These satisfy $V_S \ll V_A < V_F$. Since the fast wave is the only one that compresses the magnetic field, a motion that is highly stabilizing, all low-β tokamak ideal MHD instabilities are associated with the slow wave and the Alfvén wave. The plasma will ''slip through'' the background field rather than compress it. However it is the fast wave that sets the maximum allowable time step when using an explicit time advance.

To better understand the time step restriction imposed by the presence of the fast wave, consider the timescales associated with the three types of waves. The slow wave and Alfvén wave only propagate in the direction parallel to the background magnetic field whereas the fast wave's propagation is nearly isotropic [2]. If we denote the local safety factor as q and the local aspect ratio as ε, then the ratio of the transit times of these three waves is: $q\varepsilon^{-2}$: $q\varepsilon^{-1}$: 1. However, the difference in the explicit time step constraint associated with the three waves is much more extreme than this. The spatial resolution requirements perpendicular to the magnetic field are much more severe than those parallel to the magnetic field, making the Courant–Friedrichs–Lewy (CFL) [6] condition associated with the fast wave much more restrictive than that