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Density models for streamer discharges: Beyond cylindrical symmetry and homogeneous media

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ABSTRACT

Streamer electrical discharges are often investigated with computer simulations of density models (also called reaction-drift-diffusion models). We review these models, detailing their physical foundations, their range of validity and the most relevant numerical algorithms employed in solving them. We focus particularly on schemes of adaptive refinement, used to resolve the multiple length scales in a streamer discharge without a high computational cost. We then report recent results from these models, emphasizing developments that go beyond cylindrically symmetrical streamers propagating in homogeneous media. These include interacting streamers, branching streamers and sprite streamers in inhomogeneous media.

1. Introduction

Streamers [1–4] are transient electrical discharges that appear when a non-conducting medium is suddenly exposed to a high electric field, often localized around a high-potential pointed electrode. While the average background field might be too low for plasma generation by electron impact on neutral molecules, the streamer discharge channel can enhance the electric field at its growing tip so strongly that it can create additional plasma and propagate nevertheless. The streamer achieves this through a very nonlinear dynamics with an intricate inner structure, locally very steep density gradients and space charge concentrated in very thin layers. This structure generates the local field enhancement and maintains propagation. Therefore, the accurate simulation of single, cylindrically symmetric streamer channels is far from trivial, even if the charged species are approximated by their densities as is usually done.

It should be noted that the full process evolves on even more scales. On the one hand, when the electric field ahead of the streamer head is very high, single electrons can run away and generate hard radiation through Bremsstrahlung. To describe these effects, single electrons have to be followed in a particle model; numerical methods to track these particles in an efficient manner and the derivation of density models are discussed in this special issue by Li et al. [5]. On the other hand, in the laboratory, in technological applications and in lightning related processes, hundreds to ten-thousands of streamer channels can propagate next to each other. In this case it is vital to develop models at a more macroscopic scale than the density approximation. Through model reduction, one can derive moving boundary models [6–10] for the underlying ionization fronts, or one even can try to develop models for the streamer channel as a whole.

Within the current short review, we discuss the justification for density models for streamers, the numerical solution strategy, and then some results that go beyond single cylindrically symmetric streamers in homogeneous media. We treat interacting streamers, first through the trick of considering an infinite array of identical streamers next to each other, then

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through full 3D simulations. Next, we discuss streamer branching in full 3D. Finally we discuss how to simulate the emergence of sprite streamers from the Earth's ionosphere; a peculiar issue is here how to combine the very different scales of the electric fields generated by the thundercloud-ground-ionosphere system with the fine inner structure of the discharge and with the density of the atmosphere, that varies over a length-scale of about 7 km.

With the density models, here we focus on the oldest and most extensively investigated family of numerical streamer models; they are also known as fluid models, continuous models or reaction-drift-diffusion models.

2. Model formulation

2.1. The density model

A classical density model for a streamer discharge has the structure of a reaction-drift-diffusion equation for the electrons and reaction equations for various ions and excited species coupled to the electric field

$$\frac{\partial n_e}{\partial t} = \nabla \cdot (n_e \mu_e \mathbf{E}) + \nabla \cdot D_e \nabla n_e + S_e^{im} + S_e^{ph}, \tag{1a}$$

$$\frac{\partial [Z_i]}{\partial t} = S_i^{im} + S_i^{ph}, \quad i = 1, \ldots, N, \tag{1b}$$

$$\epsilon_0 \nabla \cdot \mathbf{E} = q, \quad \mathbf{E} = -\nabla \phi. \tag{1c}$$

Here n_e is the electron number density and $[Z_i]$ is the density of the heavy species Z_i denoting a positive or negative ion or an excited state, and μ_e and D_e are mobility and diffusion coefficients of the electrons. $S_{e,i}^{im}$ denote mostly local generation or loss of species due to reactions at direct encounter of particles; the most prominent example is electron impact ionization, i.e., the generation of electron–ion pairs through impact of electrons on neutrals; the efficiency of this process strongly depends on the electron energy that in turn is determined by gas density and electric field. $S_{e,i}^{ph}$ denote the generation of the species through radiative transport in a generically nonlocal process; the most prominent example in streamer physics is the generation of electron-O_2^- pairs through photo-ionization in air. Finally, $\mathbf{E}$ is the electric field, ϕ the electric potential, and q is the local space charge, determined as $q = \sum_i q_i\, [Z_i] - \mathrm{e}\, n_e$; here e is the elementary charge and $q_i = -\mathrm{e}, 0, +\mathrm{e}$ is the charge of species Z_i.

2.2. The underlying approximations

We list here the main approximations underlying this model. For a more thorough derivation of the model, we refer to [5,11,12] and references therein.

1. The electrons are much more mobile than the ions, and the dynamics takes place on their time scale, therefore ion motion is neglected—the validity of this approximation often has to be checked after the longest relevant time scales are found through a simulation. This approximation saves computing time and removes unnecessary complexity but it is not essential: ion mobility can be easily incorporated in streamer numerical models.
2. The electron motion is approximated by drift and diffusion within the local field. This entails that the electrons rapidly relax to an equilibrium velocity distribution where the acceleration by the electric field exactly cancels the momentum losses due to collisions with other particles, and that spatial or temporal variations of the electric field are not important. The accuracy of the drift-diffusion approximation and the local field approximation for electron currents in streamer ionization fronts was verified in [12] for electric fields up to some threshold; this statement holds for arbitrary gas density once the threshold electric field is rescaled accordingly. However, when the field at the streamer head exceeds a limit (180 kV/cm in air at standard temperature and pressure (STP) [5,11,13,14]), some electrons in the tail of the electron energy distribution function might not relax to a steady velocity anymore, but run away with a field dependent probability. Above the thermal runaway electric field (260 kV/cm in STP air [15–17]) the bulk of the electrons will keep accelerating up to relativistic energies and the drift approximation breaks down.
3. The field is calculated with electrostatic approximation. For typical current densities and diameters of streamers [18], this approximation is justified.
4. The ensembles of electrons and ions can be treated in density approximation. The validity of this approximation depends first on the gas density, and second on the specific region within the streamer. First, streamers in different gas densities n are related through similarity relations or Townsend scaling in very good approximation [18,19]; these relations imply that the electron density n_e in a streamer scales as n^2, and that the intrinsic length scales scale as $1/n$. Consequently, the total number of electrons in similar parts of streamers scales as $1/n$. This implies that the density approximation in all regions of the streamer becomes better when the gas density decreases. On the contrary, the density approximation for electrons and ions becomes worse when the density of the neutral medium increases, and it has to be reconsidered at the high densities of liquids.[1] A review of validity and corrections to these similarity solutions is given in [18].

[1] Other approximations to be reconsidered in streamers at liquid densities are the absence of heating and the neglect of electron–electron and electron–ion collisions in the source terms S^{im}. In gases, only electron-neutral collisions are included in typical streamer models, but the degree of ionization and therefore the relative frequency of electron–electron or electron–ion collisions increases with increasing density because $n_e/n \propto n$.