Journal of Computational Physics 238 (2013) 141–153

Contents lists available at SciVerse ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Auxiliary variables for 3D multiscale simulations in heterogeneous porous media

Andreas Sandvin [a,b], Eirik Keilegavlen [a,*], Jan M. Nordbotten [a]

[a] Department of Mathematics, University of Bergen, Johannes Brunsgate 12, 5008 Bergen, Norway
[b] Centre for Integrated Petroleum Research, University of Bergen, Allegaten 41, 5007 Bergen, Norway

ARTICLE INFO

Article history:
Received 9 November 2011
Received in revised form 21 November 2012
Accepted 4 December 2012
Available online 29 December 2012

Keywords:
Multiscale methods
Control volume methods
Porous media
Preconditioning
Domain decomposition

ABSTRACT

The multiscale control-volume methods for solving problems involving flow in porous media have gained much interest during the last decade. Recasting these methods in an algebraic framework allows one to consider them as preconditioners for iterative solvers. Despite intense research on the 2D formulation, few results have been shown for 3D, where indeed the performance of multiscale methods deteriorates. The interpretation of multiscale methods as vertex based domain decomposition methods, which are non-scalable for 3D domain decomposition problems, allows us to understand this loss of performance.

We propose a generalized framework based on auxiliary variables on the coarse scale. These are enrichments of the coarse scale, which can be selected to improve the interpolation onto the fine scale. Where the existing coarse scale basis functions are designed to capture local sub-scale heterogeneities, the auxiliary variables are aimed at better capturing non-local effects resulting from non-linear behavior of the pressure field. The auxiliary coarse nodes fits into the framework of mass-conservative domain-decomposition (MCDD) preconditioners, allowing us to construct, as special cases, both the traditional (vertex based) multiscale methods as well as their wire basket generalization.

1. Introduction

Geological porous media are typically characterized as heterogeneous at virtually every scale. This reflects the process by which geological formations are created, where natural sedimentation processes spanning kilometers horizontally and millennia in time lead to composite materials that are intrinsically complex in structure. Compounding the difficulties introduced by multiscale geological parameterizations are the strongly non-linear equations that describe multi-phase flow in porous media. These equations lead to challenges that are manifested in discontinuous solutions as well as both gravitational and viscous instabilities. Such phenomena are frequently best understood as multiscale in nature. As a consequence of the complexity in modeling multi-flow in geological porous media, virtually every text-book on the subject address issues of scale. We refer to [1] for classical examples.

Two main avenues are typically followed when confronting multiscale phenomena. The most classical approach, multiscale modeling, is to manipulate equations defined at a finest, verified scale, and attempt to derive effective equations valid on coarser scales. These equations are typically stated for derived variables. These derived variables broadly fall into three categories: conserved (extensive) quantities, auxiliary (intensive) state variables, and problem specific variables. This final category of variables may be unique to the problem, or to the coarser scales, and can be interpreted to represent emerging properties of the system. In some cases these emerging properties are parameterizations of what would otherwise be seen as

* Corresponding author.
E-mail addresses: Andreas.Sandvin@uni.no (A. Sandvin), Eirik.Keilegavlen@uib.no (E. Keilegavlen), Jan.Nordbotten@math.uib.no (J.M. Nordbotten).

0021-9991/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.jcp.2012.12.016

hysteretic, or non-unique, behavior. In the context of multi-phase flow in porous media, component masses are conserved at all scales, pressure is an intensive state variable at all scales, and finally various parameterizations of hysteresis or dynamical behavior are introduced to make the models appropriate in practice [2]. This classical approach has seen several formalizations in recent years, among the most instructive of which is the Heterogeneous Multiscale Method [3].

A more recent approach to handling multiscale characteristics is through adaption of the numerical methods themselves. Classically introduced as generalized finite elements by Babuska et al. [4], it was first made into a useful concept through the residual-free bubble methods [5], where multiscale features of the solution can be handled. Later, this concept was also applied to multiscale coefficients, in what is termed multiscale finite element and multiscale finite volume methods (see [6] for an introduction). While multiscale numerical methods have shown good properties on academic problems, they often fail to live up to their promise on real problems [7]. By exploiting the link between multiscale numerical methods and domain decomposition (DD), multiscale control volume methods can be framed in an iterative setting which greatly increases the potential for robust implementations [8]. However, an improved multiscale representation without iterations is still the ultimate goal.

In this paper, we propose to enhance the common understanding of multiscale numerical discretizations through an analogy to multiscale modeling. In particular, as multiscale control volume methods inherently discretize conserved quantities, it is natural to ask if the discrete approximation, like its modeling counterpart, can be enhanced through introducing problem-specific additional variables. We term these additional variables auxiliary, and the remainder of the paper is devoted to developing and verifying this concept. In particular we consider the issue of assigning boundary conditions to the local problems based on the state in the coarse variables. This poses challenges for multiscale numerical methods, especially in the presence of long correlation pathways that render non-local dependence of the solution. The problem is difficult already in two spatial dimensions, where the state in the coarse variables must be mapped onto a 1D boundary. Strategies proposed to remedy the situation include oversampling [9,10], utilizing global information [10,11] and using specialized boundary conditions [7,12]. The situation becomes worse in three spatial dimensions, since a mapping to a 2D boundary is needed. In this paper, auxiliary coarse variables are used to address these challenges. By exploiting links between multiscale control volume methods and domain decomposition, the auxiliary variables can easily be introduced in the linear solver. We consider grids with relatively few primary coarse variables (corresponding to aggressive coarsening), and enhance the coarse space by auxiliary variables. Thus the number of internal boundaries decreases, while there is enough degrees of freedom in the coarse space to capture details in the solution. Our numerical experiments involve model problems as well as industrial benchmark data. The results show that auxiliary coarse variables can improve the performance of the linear solver considerably.

The rest of the paper is structured as follows: in Section 2, multiscale methods for three-dimensional problems are discussed and difficulties are pointed out. A multiscale linear solver is introduced in Section 3, and the extension to coarse spaces is introduced in Section 4. Simulation results are presented in Section 5, and the paper is concluded in Section 6.

2. Challenges of 3D multiscale elliptic problems

In this study we consider the following elliptic problem for flow in three dimensional porous media,

$$-\nabla \cdot (\mathbf{K}\nabla u) = q, \tag{1}$$

where $\mathbf{K}$ is the permeability of the medium, u is the potential and q represents the source terms of the system. The heterogeneous structure of porous rocks is reflected in the permeability $\mathbf{K}$, which can vary by several orders of magnitude on different scales. It is the variation of this parameter which represents the major challenge, and has been the main focus of the multiscale methods for problems involving flow in porous media. Hou and Wu [13] showed that the sub-scale information of the elliptic operator can be captured within a few coarse-scale basis functions, which increases the accuracy of the recovered fine-scale solution. Several multiscale numerical methods have later been developed for the capturing of sub-scale information into local basis-functions. We refer to [6] for an overview of these methods.

A special focus of this paper will be on problems involving long correlation lengths of the parameter $\mathbf{K}$, e.g. fractures, faults and channels which occupy several coarse-scale grid blocks. The discretization of such problems are particularly difficult to upscale, and local iterations are required to guarantee accurate solutions [14]. Due to the difficulties involved, the primary focus of previous works has been the 2D problem. In the remainder of this section we will briefly discuss some of the existing challenges of multiscale numerical methods, and highlight some of the main challenges of extending these methods to 3D simulations.

2.1. Multiscale numerics

In general porous media there are rarely only two or a couple of distinct scales, but rather a continuum of physical scales which needs to be taken into account. However, for practical purposes we need to define a finest (geological) scale for the discretization of our problem. Usually the fine-scale discretization leads to a large coupled problem which is extensive and often too computationally expensive to solve. For multiscale methods, one or a couple of coarse scales are added to speed up the calculation of a fine-scale conservative solution.