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Wavelet-based adaptive large-eddy simulation with explicit filtering

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ABSTRACT

Stochastic coherent adaptive large-eddy simulation is a novel approach to the numerical simulation of turbulence, where the coherent energetic eddies are solved for, while modeling the influence of the less energetic coherent/incoherent background flow. The formal separation between resolved and unresolved field is obtained by wavelet threshold filtering that is inherent to the adaptive wavelet collocation numerical method. A new explicit wavelet filtering strategy is introduced and tested, by considering two different filtering levels: the physical level, which controls the turbulence model, and the numerical level that is responsible for the accuracy of numerical simulations. The theoretical basis for wavelet-based adaptive large-eddy simulation with explicit filtering and consistent dynamic modeling is given. Numerical experiments are presented for unsteady homogeneous turbulence, demonstrating the existence of grid-independent solutions.

1. Introduction

The stochastic coherent adaptive large-eddy simulation (SCALES) method is a novel approach to the numerical simulation of turbulent flows, where the more energetic coherent eddies are solved for, while discarding the dynamics of the less energetic background flow [1]. The decomposition of turbulence into resolved coherent and residual coherent/incoherent motions is obtained through the application of wavelet threshold filtering (WTF). The space–time evolution of the coherent velocity field is governed by the wavelet-filtered Navier–Stokes equations, where, similarly to any other large-eddy simulation (LES) approach, the effect of the unknown residual stresses is modeled.

In order to solve the SCALES governing equations in a computationally efficient manner, the adaptive wavelet collocation method (AWCM) is used, e.g. [2]. The method is a variable high-order finite-difference numerical simulation that exploits the same wavelet filtering procedure to automatically adapt the computational grid to the solution, in both location and scale. The choice of a wavelet thresholding level for the grid adaptation automatically introduces a formal separation between eddies that are resolved by the wavelet collocation grid and unresolvable sub-grid eddies. Up to now, the filtering effect induced by the use of the AWCM has been used to define the filtered-velocity in the SCALES approach, without distinguishing between physical (model) filtering and numerical filtering that is related to truncation of wavelet basis and corresponding computational mesh. Thus, there has been no practical reason, so far, to discern between explicit and implicit wavelet-filtering. Along this line, the residual stresses have been referred to and treated as subgrid-scale (SGS) stresses, e.g. [3,4].

The implicit approach makes it impossible to separate between filtering, modeling and numerical issues. This is a fundamental problem in LES methods [5] that becomes even more important in the wavelet-based formulation, where the numerical solver allows for the automatic mesh refinement in flow regions with inadequate modeled dissipation.

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Furthermore, SCALES method with implicit filtering is particularly sensitive to numerical errors since the choice of a relatively high thresholding level unavoidably affects the accuracy of the AWCM-based solution.

In this work, following a strategy that is sometimes adopted in classical non-adaptive LES, e.g. [6,7], a new wavelet-based adaptive LES method with explicit filtering is introduced and tested. Two different filtering levels are considered: the physical level, which controls the level of turbulence model, and the numerical level that is solely responsible for the accuracy of calculations. The superposition of explicit filtering allows to practically control the computational errors, while keeping apart the filtering and the numerical issues.

The theoretical basis for SCALES with explicit filtering is given and numerical experiments are carried out for freely decaying homogeneous incompressible turbulence. The use of explicit wavelet filtering is studied in order to address the potential of the method to obtain grid-independent numerical solutions, as found for classical LES [8].

In the next section, the wavelet-based adaptive LES approach is briefly reviewed. The new explicit filtering SCALES method is introduced in Section 3, along with the consistent dynamic procedure for modeling the residual stresses. In Section 4, the numerical experiments are presented and discussed. Finally, in Section 5, some concluding remarks are made.

2. Wavelet-based adaptive large-eddy simulation

Different numerical methods for solving the Navier–Stokes equations in adaptive wavelet bases have been recently developed, by exploiting the efficient wavelet decomposition of turbulence into space-scale contributions, e.g. [9–12]. In the following, the wavelet-based LES approach is concisely reviewed, along with the dynamic procedure for modeling the effect of residual motions.

2.1. Wavelet-filtered velocity

In the wavelet-based adaptive LES approach, the turbulent velocity field is decomposed into two different parts: a coherent more energetic velocity field and a residual less energetic coherent/incoherent one. WTF is performed by applying the wavelet-transform to the unfiltered field, zeroing the wavelet coefficients below a given threshold, say, ϵ, and transforming back to the physical space [1]. Formally, the wavelet-filtered velocity, $\overline{u_i}^{>\epsilon}(\mathbf{x})$, is defined by expressing the instantaneous velocity field in terms of wavelet basis functions and retaining only significant wavelets, e.g. [13]:

$$\overline{u_i}^{>\epsilon}(\mathbf{x}) = \sum_{\mathbf{l}\in\mathcal{L}^0} c^0_{\mathbf{l}}\phi^0_{\mathbf{l}}(\mathbf{x}) + \sum_{j=0}^{+\infty}\sum_{\mu=1}^{2^n-1} \sum_{\substack{\mathbf{k}\in\mathcal{K}^{\mu,j} \\ |d^{\mu,j}_{\mathbf{k}}| > \epsilon\|u_i\|_{\mathrm{WTF}}}} d^{\mu,j}_{\mathbf{k}}\psi^{\mu,j}_{\mathbf{k}}(\mathbf{x}). \tag{1}$$

In the above decomposition, bold subscripts denote n-dimensional indexes, while $\mathcal{L}^0$ and $\mathcal{K}^{\mu,j}$ are n-dimensional index sets associated with scaling functions at zero level of resolution ($\phi^0_{\mathbf{l}}$) and wavelets of family μ and level j ($\psi^{\mu,j}_{\mathbf{k}}$), respectively. Each level of resolution j consists of a family of wavelets $\psi^{\mu,j}_{\mathbf{k}}$ having the same scale but located at different grid positions.

Thus, the turbulent velocity field results decomposed as $u_i = \overline{u_i}^{>\epsilon} + u_i'$, where $\overline{u_i}^{>\epsilon}$ stands for the wavelet-filtered velocity at level ϵ, while u_i' corresponds to the background less energetic flow. Depending on the choice of the WTF level that is dictated by the desired turbulence resolution, only a relatively small number of wavelets are retained in representing the filtered field $\overline{u_i}^{>\epsilon}$. It can be shown that the relative difference between unfiltered and filtered velocity is bounded by the thresholding level ϵ [14].

The high compression property of the wavelet-based decomposition is illustrated in Table 1, where the percentage of active wavelets and retained energy/enstrophy are reported as a function of the WTF level ϵ, for a given turbulent velocity field. The latter corresponds to an instantaneous realization of statistically stationary homogeneous turbulence at $Re_\lambda \cong 120$ (λ being the Taylor microscale) provided by a pseudo-spectral DNS calculation [15]. For instance, by retaining less than 1% of the 512^3 available wavelets, one is able to capture more than 99% of the total energy and almost 90% of the total enstrophy of the flow (as it happens for $\epsilon = 0.25$).

For a given level of turbulence resolution, the fraction of enstrophy that is resolved is lower than the energy fraction because the small dissipative scales are partially filtered out. However, differently from classical low-pass filtering used in non-adaptive LES, the small scale turbulence is partly represented by the wavelet-filtered field. In fact, one of the distinctive features of WTF stands in the ability to capture energetic coherent eddies of any size. This is particularly evident

Table 1
Percentage of active wavelets and retained energy/enstrophy for different WTF levels.

Level ϵ	Wavelets (%)	Energy (%)	Enstrophy (%)
0.45	0.11	97.8	66.0
0.35	0.26	99.0	78.4
0.25	0.61	99.5	87.0
0.15	1.86	99.9	95.1