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Short Note

A novel outflow boundary condition for incompressible laminar wall-bounded flows

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1. Introduction

The aim of this note is to propose an alternative to the classical convective outlet boundary condition heavily used for wall-bounded laminar/turbulent flows.

Despite the huge improvements in computational power in the last few years, numerical simulations (DNS or LES) of boundary layers are still expensive and any simulation may take many weeks of processing time to achieve statistically stationary results. Hence, any methodology that reduces the computational time is welcome. Concerning the outflow boundary, many conditions can be applied, such as a streamwise periodicity [11] or a zero-gradient condition on the velocity [6]. However, in most of the studies that deal with numerical simulations of a turbulent flow, a convective condition is used as the exit boundary condition. This exit condition is a solution to the linearised convective equation (Eq. (1)), where the convection velocity U_c is chosen to be either the maximum streamwise velocity [8], the mean streamwise velocity at this plane [4,7] or the local velocity [2,5] and x is the streamwise direction:

$$\frac{\partial u_i}{\partial t} + U_c \frac{\partial u_i}{\partial x} = 0 \qquad (1)$$

In the case of a boundary layer, however, the implementation of Eq. (1) can create a singularity in the shear stress distribution. The shear stress error in the context of a fractional step algorithm [3] will lead to an overcorrection of the velocity field by the pressure, which will contaminate a large part of the computational domain because of the elliptic nature of the Poisson equation. Moreover, the effect of this singularity contaminates an increased portion of the computation domain with decreasing Reynolds number. Fig. 1 represents the pressure field obtained for a laminar boundary layer developing over a flat plate obtained using the outflow boundary condition (Eq. (1)), U_c being the local velocity. The contamination, as made evident in the non-uniform pressure contours in the figure, extends approximately one fifth of the way into the simulation domain. The "polluted" pressure field clearly is wrong. The main consequence of having this kind of polluted

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Fig. 1. Non-dimensional pressure field for a laminar boundary layer developing over a flat plate obtained with the classical outlet condition.

zone is that the domain size is larger than it should be (with the concomitant increase in computational cost) assuming that there is no impact farther upstream and that it is acceptable to ignore this region.

The linearised form of the Navier–Stokes equations (i.e. Eq. (1)) at the outlet implies that the first- and second-order wall-normal derivatives are negligible. This may be true for some flows; however, it is demonstrated below that in the case of laminar boundary layers this is not true. The velocities are small close to the wall; however, they increase rapidly away from the wall. The magnitude of the wall-normal derivatives are, therefore, large, as can be observed in Fig. 2.

Fig. 2 demonstrates that the classical term $u_x \frac{\partial u_x}{\partial x}$ is much larger in magnitude than other terms. Moreover, for higher Reynolds numbers, the magnitude of these terms remain non-negligible (see Fig. 3). These results clearly are in accordance with the boundary layer theory since the boundary layer equations directly imply that all these terms are of the same order of magnitude [10].

Consider the following equations as boundary conditions:

$$\frac{\partial u_i}{\partial t} + u_x \frac{\partial u_i}{\partial x} + u_y \frac{\partial u_i}{\partial y} - \nu \frac{\partial^2 u_i}{\partial y^2} = 0 \tag{2}$$

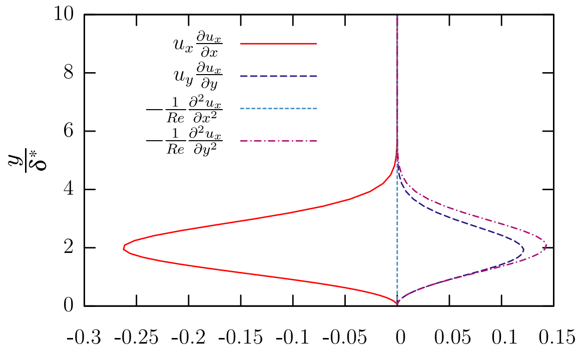

Fig. 2. Order of magnitude of the different terms in the streamwise Navier–Stokes equation for a laminar boundary layer developing over a flat plate at $Re_\theta = 320$.

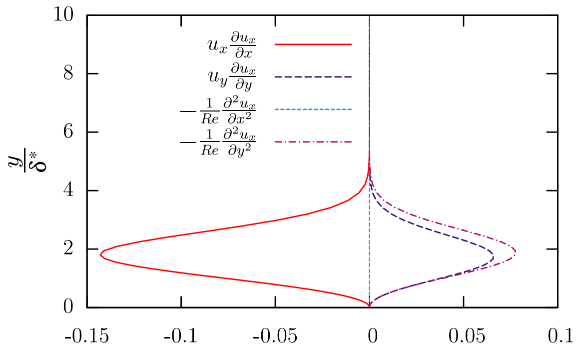

Fig. 3. Order of magnitude of different terms in the streamwise Navier–Stokes equation for a laminar boundary layer developing over a flat plate at $Re_\theta = 700$.