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A high-order finite-volume algorithm for Fokker–Planck collisions in magnetized plasmas

Z. Xiong *, R.H. Cohen, T.D. Rognlien, X.Q. Xu

Lawrence Livermore National Laboratory, Fusion Energy Program Livermore, CA 94550, USA

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Abstract

A high-order finite-volume algorithm is developed for the Fokker–Planck Operator (FPO) describing Coulomb collisions in strongly magnetized plasmas. The algorithm uses a generic fourth-order reconstruction scheme on an unstructured grid in the velocity space spanned by parallel velocity and magnetic moment. By analytically mapping between different coordinates, it produces an accurate and density-conserving numerical FPO for an arbitrary choice of velocity space coordinates. A linearized FPO in constants-of-motion coordinates is implemented as an example of the present algorithm combined with a cut-cell merging procedure. Numerical tests include the thermalization of a test distribution with a background Maxwellian at a different temperature, and the return to isotropy for a distribution initialized with a velocity space loss-cone. Utilization of the method for a nonlinear FPO is straightforward but requires evaluation of the Trubnikov–Rosenbluth potentials.

Keywords: Fokker–Planck collisions; High-order scheme; Finite volume; Constants-of-motion coordinates

1. Introduction

The differential Fokker–Planck Operator (FPO) describes the particle collisions in a fully ionized plasma through shielded electrostatic Coulomb fields. Such Coulomb collisions are important in many systems, including laboratory plasma physics devices for basic studies, magnetic and inertial fusion, industrial material processing, and astrophysics. In magnetized plasma, because of the gyro-motion of the charged particles around magnetic field lines, the FPO is typically written in spherical coordinates spanned by (v, θ, ϕ), where v is the particle speed, θ is the pitch angle and ϕ the gyro-angle. For strong magnetic fields, many physical phenomena have characteristic time scales much longer than the gyro-period, and characteristic length scales much larger than the gyro-radius. In such cases, a gyro-averaging procedure may be applied and the resulting distribution function becomes independent of gyro-angle ϕ. By further using a series expansion with Legendre polynomials $L_n(\cos\theta)$ in the θ direction, the evaluation of FPO is reduced to solving a series of one-dimen-

* Corresponding author. Present address: Schlumberger Sugar Land Technology Center. Tel.: +1 281 285 7331.
E-mail address: zxiong@gmail.com (Z. Xiong).

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sional equations of v only [1,2]. Following this approach, Chang and Cooper [3] developed a finite difference scheme that conserves particle number density. This scheme was further extended by Epperlein [4] to conserve particle energy. Khabibrakhmanov and Khazanov [5] have recently solved these equations using a spectral collocation method.

The velocity coordinates (v, θ), though convenient for evaluating the FPO, are not particularly suitable for simulating spatially inhomogeneous plasmas where particle advection, including parallel streaming and perpendicular drifts, is important. To accurately compute the particle orbits, the velocity coordinates are often determined by the particular choice of the spatial advection schemes. In such cases, the FPO needs to be evaluated in the same coordinates and its discretization becomes truly two-dimensional. Chacon et al. [6] proposed a tensor formulation for two-dimensional FPO and studied the finite difference scheme in cylindrical coordinates for improved energy conservation. Other algorithms dealing with non-isotropic, multi-dimensional FPOs use Cartesian velocity coordinates directly [7,8]. Cartesian coordinates, however, are almost never used directly for strongly magnetized plasmas owing to the usefulness of averaging over the rapid particle gyromotion to remove one dimension from the computation. Recent attempts to couple the FPO with the Vlasov equation in different velocity coordinates have ignored the spatial dependence of the distribution function in the collision operator [9,10].

The so-called constants-of-motion coordinates, e.g. the total energy E and the magnetic moment μ, have been used in gyrokinetic simulations of fusion plasma with both particle [11] and continuum [12] formulations. For these simulations, it is critical that the passing and trapped particle orbits are accurately represented. The choice of (μ, E) coordinates is advantageous because (μ, E) remain constant along particle orbits (in the absence of collisions and time-varying fields), and the velocity coordinates (μ, E) are thus orthogonal to the spatial coordinates. For instance, the collisionless Vlasov equation is particularly simple when written in (μ, E) coordinates. To compute collisional effects accurately, the same constants-of-motion coordinates should be used in the FPO as well. The approach of using direct interpolation of the collision operator between different velocity coordinates has been found unsatisfactory, particularly with respect to the conservation properties. In this paper, we present an algorithm for computing the FPO in constants-of-motion coordinates based on a generic high-order finite volume scheme on unstructured grids, which is inherently particle-number-conserving. Although focusing on the constants-of-motion coordinates in this paper, our goal is to develop a numerical FPO that is accurate, conservative and easily applied to different coordinates systems.

The strategy is to first choose a convenient but fixed coordinate system, e.g. in this case $(v_\parallel, \mu)$, and then evaluate the FPO in these coordinates using a conservative, high-order finite volume scheme on a unstructured mesh. Here $v_\parallel$ is the velocity along the magnetic field. The finite volume discretization is inherently density conserving, and an unstructured mesh decouples the choice of coordinates and the gridding strategy. In this way, different velocity coordinates can be mapped *directly* onto the chosen $(v_\parallel, \mu)$ coordinates, with a regular grid in the former typically becoming an irregular and unstructured grid in the latter. After the mapping, the solution we obtain still maintains high-order accuracy and good conservation properties. In this sense, the evaluation of the FPO is independent of the choice of velocity coordinates, and the constants-of-motion coordinate set is but one such choice. For simplicity, the method is illustrated using a linearized FPO, where the collision diffusion coefficients are known by assuming the background particle distribution to be Maxwellian. For the nonlinear FPO, the diffusion coefficients need to be obtained first by solving Trubnikov–Rosenbluth potentials [13], which is an important but rather independent problem and shall be dealt with separately. Once the diffusion coefficients are known, the algorithm described here applies to the nonlinear FPO directly.

The remainder of the paper is organized as follows: the formulation of the FPO in $(v_\parallel, \mu)$ coordinates is given in Section 2 both for nonlinear and linearized cases. The high-order finite volume scheme on a general unstructured mesh is presented in Section 3. In Section 4, we describe the cut-cell method in (μ, E) space, and the choice of stencils for finite volume reconstruction. The numerical tests are presented in Section 5, and the concluding remarks are given in Section 6.

2. Fokker–Planck collision operator

Here the general Fokker–Planck collision operator is given in $(v_\parallel, \mu)$ coordinates, followed by the linearized version about a fixed Maxwellian distribution function describing the background field particles.