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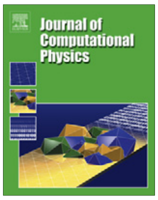

A hierarchical fracture model for the iterative multiscale finite volume method

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ABSTRACT

An iterative multiscale finite volume (i-MSFV) method is devised for the simulation of multiphase flow in fractured porous media in the context of a hierarchical fracture modeling framework. Motivated by the small pressure change inside highly conductive fractures, the fully coupled system is split into smaller systems, which are then sequentially solved. This splitting technique results in only one additional degree of freedom for each connected fracture network appearing in the matrix system. It can be interpreted as an agglomeration of highly connected cells; similar as in algebraic multigrid methods. For the solution of the resulting algebraic system, an i-MSFV method is introduced. In addition to the local basis and correction functions, which were previously developed in this framework, local fracture functions are introduced to accurately capture the fractures at the coarse scale. In this multiscale approach there exists one fracture function per network and local domain, and in the coarse scale problem there appears only one additional degree of freedom per connected fracture network. Numerical results are presented for validation and verification of this new iterative multiscale approach for fractured porous media, and to investigate its computational efficiency. Finally, it is demonstrated that the new method is an effective multiscale approach for simulations of realistic multiphase flows in fractured heterogeneous porous media.

1. Introduction

Mathematical formulations describing flow in natural porous media are typically governed by highly heterogeneous anisotropic tensorial coefficients (hydraulic conductivity) at different scales. Moreover, a considerable percentage of natural formations, e.g. carbonate reservoirs, are fractured in the sense that there exist highly conductive channels (with small apertures) at various length scales acting as phase transport highways. In addition to the complex geometries, the high contrast in the physical properties and length scales (compared to those of the matrix) results in very expensive fine scale simulations. Therefore, there have been extensive studies during the past five decades to reduce the problem complexity and as a result different modeling approaches and numerical strategies suitable for different types of fractures have been proposed [1–16].

The dual porosity model approach and its divisions [1–4] were proposed for naturally fractured porous media with many small fractures. More precisely, this method introduces effective coefficients for $(n-1)$ dimensional (D) fractures by mapping (upscaling) them into a continuum nD domain. This upscaling based strategy results in reasonably efficient simulations with the cost of additional assumptions. More specifically, this method is appropriate for problems with only small scale

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fractures. For problems with long scale fractures, however, this approach fails to provide good solutions. This is due to the fact that in this approach no general upscaling strategy is possible. Note that long fractures can be treated by dual permeability approaches.

To obtain more accurate simulations, the discrete fracture modeling approach was devised; see e.g. [5,8,13]. In that approach, geometry and locations of fractures are honored accurately by using complex unstructured gridding techniques [14]. The grid is generated with the constraints that the fracture elements are located at the matrix cell interfaces and that the matrix cells around fractures are small enough to capture the correct fracture geometries. The latter constraint often results in very small cells, especially near intersections. Besides the fact that small cells lead to big linear systems, they also impose time step restrictions for multiphase transport simulations.

It is very important to keep in mind that this approach has limited applicability for realistic scenarios due to the complex conforming grids. Moreover, this approach is not suited for dynamic fracture network problems, as e.g. in simulations of enhanced geothermal systems, where the grid has to be updated frequently due to generations of new fractures. In such applications, it is preferable to work with independent discretizations for the discrete large fractures and the damaged matrix.

Motivated by the previously discussed issues, a hierarchical fracture modeling approach was introduced [10–12]. In this approach, small scale fractures are homogenized and treated as a continuous damaged matrix with effective coefficients. Large-scale fractures, on the other hand, are explicitly represented by a coupled discrete fracture model. More precisely, simple structured nD and $(n-1)$D grids are independently generated for matrix and fractures, respectively. Note that neither grid alignment nor any other constraints apply.

In this work, a hierarchical fracture modeling approach which is suited for multiscale methods [17–31] is introduced. Moreover, as will be demonstrated later in this paper, for highly conductive fractures the proposed strategy leads to enhanced convergence rates. One additional constraint per fracture network is added to the matrix equations, which is crucial to ensure enough coupling to achieve good convergence rates. Combining a hierarchical fracture model to a muliscale method for reservoir simulation is of interest, since multiscale methods are capable of honoring fine-scale transmissibility variations with much fewer degrees of freedom (DOF) than classical simulators. Such fine-scale variations become an even bigger issue in highly fractured reservoirs. In order to properly deal with transport it is highly desirable to work with a multiscale method which delivers conservative fine-scale velocity fields; see e.g. [32,33]. Therefore, here the multiscale finite-volume (MSFV) method is favored, which requires fewer DOFs than mixed multiscale methods and still is conservative opposed to e.g. the multiscale finite-element methods. Moreoever, as shown recently, in this framework the error can systematically be reduced [34–39]; adaptive in space and time [40]. Opposed to classical iterative solvers like algebraic multigrid (AMG) [41], conservative solutions can be constructed after any iteration, i.e. it is not required to fully converge.

In this work, the i-MSFV method is extended for the solution of the hierarchical fracture problem using the proposed sequential coupling strategy. To capture fractures accurately at the coarse scales, local fracture functions are introduced. The fracture functions are solved within local problems, similar to the basis and correction functions, based on the full governing equations (with fracture-matrix couplings) subject to the reduced problem boundary conditions. This results in only one additional coarse DOF per fracture network. Inner and outer iterations are applied to enhance the correction function boundary conditions and for the convergence of the sequentially-coupled fracture-matrix system, respectively. A classical iterative fine-scale solver is used as the smoother, which is necessary to guarantee convergence. Alternatively, a Krylov sub-space method like GMRES [42] can be used to stabilize the system [37]. Here we employ line-relaxation as a smoother, since it is very robust for problems with stretched grids. The convergence history of the new i-MSFV method for fractured porous media is tested for a wide range of test cases. Moreover, the efficiency of the method is studied for multiphase flow in heterogeneous and homogeneous fractured porous media. Numerical results show that the i-MSFV method is a flexible iterative method which is efficient for multiphase simulations in highly heterogeneous fractured porous media.

The paper is organized as following. Section 2 consists of two subsections. First, the governing equations for the hierarchical fracture modeling approach together with the model parameters are introduced. Then, in the second subsection, a previously proposed tightly coupled simulation strategy is explained. The sequentially-coupled strategy is then introduced in Section 3 followed by Section 4, where the i-MSFV method for non-fractured systems is explained. The new i-MSFV method for the fractured porous media is introduced in Section 4; and numerical results for single and multiphase flow scenarios are presented in Section 5. Finally, the paper is concluded in Section 6.

2. Hierarchical fracture modeling approach

In this section the hierarchical fracture modeling approach is explained. First, the governing equations together with the model parameters, and then a previously presented fully coupled numerical simulation strategy [10,11] are explained.

2.1. Governing equations and modeling parameters

Here, Darcy's law for incompressible multiphase flow without capillary nor gravity effects is assumed. In that case total volume balance reads

$$-\nabla \cdot (\boldsymbol{K}\lambda_t \cdot \nabla p) = q_t \text{ on } \Omega \subset \mathbb{R}^n \tag{1}$$