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Test Method

A simplified compression test for the estimation of the Poisson's ratio of viscoelastic foams

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ABSTRACT

This paper describes a simplified procedure for determining the Poisson's ratio of homogeneous and isotropic viscoelastic materials. A cylindrical shaped material is axially excited by an electromagnetic shaker and consequent displacement waves are investigated. Using a frequency sweep as an excitation signal, the frequency domain displacement response is measured upstream and sideways of the sample itself. A plane cross-section analytical model of the experimental setup is used to estimate Poisson's ratio through a minimisation-based procedure, applied to radial displacement once the complex modulus has been directly determined under the assumption of spring-like behaviour of the axial displacement. The results are presented and discussed for different materials and compared to well-established quasi-static and finite element simulations.

1. Introduction

Poisson's ratio can play a relevant role in characterizing the linear dynamic behaviour of viscoelastic materials for noise and vibration control. In addition, this parameter occurs in several equations to be solved within the context of analytical and numerical (finite element method, statistical energy analysis, transfer matrix method) simulations. Poisson's ratio is defined as the ratio of lateral strain to axial strain in an axially loaded linear elastic solid, and this ratio is a real number in the case of ideal elasticity. In contrast, in viscoelastic materials, as a result of damping, this ratio can be considered as a complex number [1,2]. However, several studies [3,4] have demonstrated that a real valued and frequency-independent Poisson's ratio can provide reliable results and can be considered a good enough approximation when the aim is to calculate main vibro-acoustical indicators (dynamic stiffness, sound absorption, sound transmission loss, etc …).

In literature, several methods (direct and indirect) have been proposed for determining mechanical parameters of materials for vibration and noise control applications, and a comprehensive review is discussed in Ref. [5]. As stated in Ref. [2], while a lot of research has been proposed for the measurement of complex moduli, fewer experimental works have focused on determining Poisson's ratio. Recently, methods based on digital image correlation (DIC) through uniaxial relaxation tests [6] and empirical correlation between hardness and elastic moduli, along with the usual instrumented indentation test [7], have been proposed in literature for characterizing the Poisson's ratio of polymeric and composite materials. Although both methods can give a reliable estimation of Poisson's ratio, they have been applied to material having a hardness which is too high if compared with foams used in noise and vibration control applications.

The aim of this research is to present a method to determine Poisson's ratio (real valued and frequency independent), through measuring the radial displacement of a cylinder of homogeneous and isotropic material at low frequencies, once the complex modulus has been determined in advance, using a transfer matrix approach, as described in Ref. [8]. In particular, an analytical model for axial and radial displacement, based on the Mindlin-Hermann two modes theory [9], has been applied and an estimation of Poisson's ratio can be easily obtained by minimizing the difference between experimental and numerical radial displacement in the frequency domain. Measurement and analyses are limited in a frequency range where all tested samples are much smaller than the longitudinal wavelength.

The paper is organised as follows. Section 2 contains a description of methodology. A description of the experimental set-up and tested materials is provided in Section 3. Section 4 contains analytical model validation, results obtained using the proposed

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methodology and a comparison of different measurement techniques. The last section contains concluding remarks.

2. Description of methodology

2.1. Theoretical background

Consider a solid, elastic, isotropic cylinder of finite length L and radius R, as shown in Fig. 1. The cylinder is assumed to be exited in $z = 0$ with a unit displacement in z coordinate (that is $u\,(z = 0, r) = 1$ and $w\,(z = 0, r) = 0$), and it is free to vibrate elsewhere.

Assuming axisymmetric excitation and, therefore, the response of the cylinder and harmonic dependency on time (i.e. $e^{i\omega t}$), the dynamic equilibrium equation can be expressed in cylindrical coordinates (r, θ, z) as:

$$\begin{aligned} &\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\sigma_{rz} = -\rho\omega^2 u \\ &\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = -\rho\omega^2 w \end{aligned} \tag{1}$$

where ρ [kg/m^3] is the material density, ω [rad/s] is the angular frequencies and σ_{rr}, σ_{zz}, $\sigma_{\theta\theta}$ and σ_{rz} are the normal and shear components of the stress tensor. Displacement in a tangential direction can be neglected, due to the axial symmetry of the problem, meaning that no torsional vibration is present.

According to the Mindlin-Hermann (plane cross-section) theory [9], the axial and radial displacement can be defined as:

$$\begin{aligned} u(r,z) &= u_0(z) \\ w(r,z) &= r \cdot u_1(z) \end{aligned} \tag{2}$$

which correspond to the first order approximation of a power series expansion, as discussed in Ref. [10].

Under such assumptions, it is straightforward to demonstrate that functions u_0 and u_1 can be found by solving the following set of partial differential equations:

$$\begin{aligned} &-\left(\rho\omega^2 u_0 + (\lambda + 2\mu)\frac{\partial^2 u_0}{\partial z^2}\right) - 2\lambda\frac{\partial u_1}{\partial z} = 0 \\ &2\lambda S\frac{\partial u_0}{\partial z} - I_2\left(\rho\omega^2 u_1 + \mu\frac{\partial^2 u_1}{\partial z^2}\right) + 4S(\lambda + \mu)u_1 = 0 \end{aligned} \tag{3}$$

where $S = 4\pi R^2$ is the surface area of the cross-section, $I_2 = \pi R^4/2$ is the polar moment of inertia of the cross-section, λ and μ are the lame coefficients:

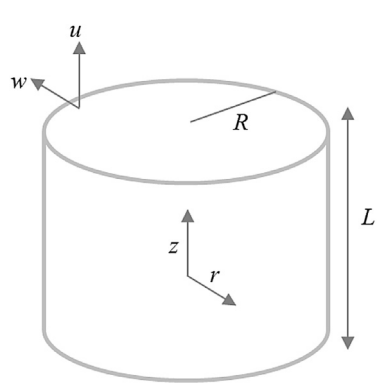

Fig. 1. A solid cylinder of length L and radius R. u and w are the axial and radial components of displacement, respectively.

$$\lambda = \frac{E\nu}{(1-2\nu)(1+\nu)} \quad \mu = \frac{E}{2(1+\nu)} \tag{4}$$

E [Pa] and ν [−] being the elastic complex modulus and Poisson's ratio, respectively.

The set of differential equations Eq. (3) can be solved applying the boundary conditions described here above, that can be expressed in terms of function u_0 and u_1 as follows [10]:

$$\begin{aligned} &u_0(z)|_{z=0} = 1\,, \quad u_1(z)|_{z=0} = 0 \\ &\left[(\lambda + 2\mu)\frac{\partial u_0(z)}{\partial z} + 2\lambda u_1(z)\right]\Big|_{z=L} = 0\,, \quad \frac{\partial u_1(z)}{\partial z}\Big|_{z=L} = 0 \end{aligned} \tag{5}$$

The reliability of the proposed analytical model will be verified against finite element simulations in Section 4A.

2.2. Methodology

In real experimental tests, the material is mounted on an aluminium support plate which is excited by an electromagnetic shaker in the z direction. Consequently, an imposed displacement (or velocity) is applied to the bottom side of the material while remaining surfaces are free to vibrate.

Using a logarithmic sine sweep as the excitation signal, the axial and radial velocity responses v_1(t) in (z, r)=(L, 0) and v_2(t) in (z, r)=($L/2$, R) are determined using a laser vibrometer, as shown in Fig. 2.

Assuming time harmonic behaviour of the measured quantities and a unit input displacement, from the experimental tests, it is possible to calculate axial and radial displacements in the frequency domain as follows:

$$\begin{aligned} &U_{\exp}(\omega)\big|_{\substack{z=L\\ r=0}} = \frac{V_1(\omega)}{j\cdot\omega} \quad [m] \\ &W_{\exp}(\omega)\big|_{\substack{z=L/2\\ r=R}} = \frac{V_2(\omega)}{j\cdot\omega} \quad [m] \end{aligned} \tag{6}$$

where $V_1(\omega)$[m/s] and $V_2(\omega)$[m/s] are the complex frequency spectra calculated by applying a Fourier transform to measured velocity signals in the time domain.

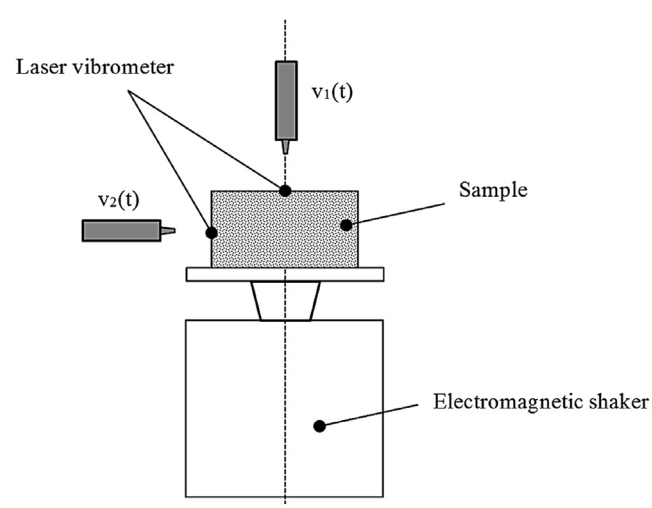

Fig. 2. Measurement layout.