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Absorbing boundary conditions for nonlinear Euler and Navier–Stokes equations based on the perfectly matched layer technique

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Abstract

Absorbing boundary conditions for the nonlinear Euler and Navier–Stokes equations in three space dimensions are presented based on the perfectly matched layer (PML) technique. The derivation of equations follows a three-step method recently developed for the PML of linearized Euler equations. To increase the efficiency of the PML, a pseudo mean flow is introduced in the formulation of absorption equations. The proposed PML equations will absorb exponentially the difference between the nonlinear fluctuation and the prescribed pseudo mean flow. With the nonlinearity in flux vectors, the proposed nonlinear absorbing equations are not formally perfectly matched to the governing equations as their linear counter-parts are. However, numerical examples show satisfactory results. Furthermore, the nonlinear PML reduces automatically to the linear PML upon linearization about the pseudo mean flow. The validity and efficiency of proposed equations as absorbing boundary conditions for nonlinear Euler and Navier–Stokes equations are demonstrated by numerical examples.

Keywords: Non-reflecting boundary condition; Perfectly matched layer; Navier–Stokes equations; Euler equations; Computational fluid dynamics; Computational aeroacoustics

1. Introduction

Non-reflecting boundary condition is a critical component in the development of computational fluid dynamics (CFD) and computational aeroacoustics (CAA) algorithms. It remains a significant challenge particularly for problems involving nonlinear governing equations. Perfectly matched layer (PML) is a technique of developing non-reflecting boundary conditions by constructing matched equations that can absorb outgoing waves at open computational boundaries. It was originally designed for computational electro-magnetics [5,6,8,28,27,7]. The significance of the PML technique lies in the fact that the absorbing zone is theoretically

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reflectionless for multi-dimensional linear waves of any angle and frequency. In the past few years, substantial progress has been made in the development of the PML technique for the Euler equations, starting with the studies for cases with constant mean flows, followed by extensions to cases with non-uniform mean flows [16,17,14,1,18,19,4,10,9]. Most recently, applications of PML to linearized Navier–Stokes equations and nonlinear Navier–Stokes equations have been discussed in [12,13]. A recent progress review is given in [21].

Although the PML technique itself is relatively simple when it is viewed as a complex change of variables in the frequency domain, it is important to note that, for the PML technique to yield stable absorbing boundary conditions, the phase and group velocities of the physical waves supported by the governing equations must be consistent and in the same direction [3,4,9,18]. For governing equations that support physical waves of inconsistent phase and group velocities, such as the Euler or Navier–Stokes equations for fluid dynamics, a space–time transformation may be required before applying the PML technique in the derivation process [18,19]. This space–time transformation corrects the inconsistency in the phase and group velocities and thus permits the application of the PML technique. An emerging method of formulating PML involves essentially three steps [21]:

1. A proper space–time transformation is determined and applied to the governing equations.
2. A PML complex change of variables is applied in the frequency domain.
3. The time domain absorbing boundary condition is derived by a conversion of the frequency domain equations.

This procedure has been successfully applied to the derivation of PML for the linearized Euler equations in [18,19].

In this paper, further application of the PML technique to the nonlinear Euler and Navier–Stokes equations is considered. Derivation of the absorbing equation is proceeded by applying the three steps outlined above to the nonlinear Navier–Stokes equations, which include the Euler equations as a special case. However, unlike the PML for linear equations, the conversion to time domain equations does not result in formally perfectly matched equations due to the nonlinearity in flux vectors. Nonetheless, the proposed absorbing equations are still effective for nonlinear problem as we will show in numerical examples. Furthermore, the nonlinear PML reduces automatically to the linear PML upon linearization. The current formulation offers a natural extension of the linear PML to nonlinear equations. For convenience of implementation in most existing CFD and CAA codes, all PML equations are formulated for the governing equations in the conservation form.

To absorb the nonlinear disturbances, a concept of "pseudo mean flow" is introduced. This makes the PML possible without knowing the exact mean flow at the start of the computation. Equations are derived that absorb the difference between the pseudo mean flow and the nonlinear disturbances, including the vorticity, acoustic, and entropy waves. One limitation of the current paper is that the pseudo mean flow is assumed to be aligned with one of three spatial axes. Recent efforts and new developments on extending the PML for oblique mean flows can be found in [11,2,24].

The rest of the paper is organized as follows. In the next section, the PML absorbing boundary condition is derived for the nonlinear Navier–Stokes equations. Further discussions on the formulation are given in Section 3. In Section 4, numerical examples that validate the effectiveness and stability of the PML for nonlinear Euler and Navier–Stokes equations will be presented. They include the absorption of a convective isentropic vortex in compressible flows, shear flow vortices and vortices shedded from flow over a circular cylinder, calculation of flat plate boundary layers, and propagation of a 3D acoustic pulse. Concluding remarks are given in Section 5.

2. Derivation of PML equations for nonlinear Navier–Stokes equations

2.1. Governing equations

We consider the three-dimensional compressible nonlinear Navier–Stokes equation written in the conservation form as