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Electric and magnetic losses modeled by a stable hybrid with explicit–implicit time-stepping for Maxwell's equations

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Abstract

A stable hybridization of the finite-element method (FEM) and the finite-difference time-domain (FDTD) scheme for Maxwell's equations with electric and magnetic losses is presented for two-dimensional problems. The hybrid method combines the flexibility of the FEM with the efficiency of the FDTD scheme and it is based directly on Ampère's and Faraday's law. The electric and magnetic losses can be treated implicitly by the FEM on an unstructured mesh, which allows for local mesh refinement in order to resolve rapid variations in the material parameters and/or the electromagnetic field. It is also feasible to handle larger homogeneous regions with losses by the explicit FDTD scheme connected to an implicitly time-stepped and lossy FEM region. The hybrid method shows second-order convergence for smooth scatterers. The bistatic radar cross section (RCS) for a circular metal cylinder with a lossy coating converges to the analytical solution and an accuracy of 2% is achieved for about 20 points per wavelength. The monostatic RCS for an airfoil that features sharp corners yields a lower order of convergence and it is found to agree well with what can be expected for singular fields at the sharp corners. A careful convergence study with resolutions from 20 to 140 points per wavelength provides accurate extrapolated results for this non-trivial test case, which makes it possible to use as a reference problem for scattering codes that model both electric and magnetic losses.

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1. Introduction

Radar absorbing materials (RAM) with electric and magnetic losses are important for the reduction of the radar cross section (RCS) in stealth applications [1]. Optimized designs may involve geometries with sharp

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edges and inhomogeneous materials. Thus, *local* mesh refinement may be necessary to resolve rapid variations in the electromagnetic field and/or the material parameters.

Ampère's law with electric losses modeled by an electric conductivity and Faraday's law with a magnetic conductivity that models the magnetic losses can be time-stepped as a system of first order differential equations. Several electromagnetic field solvers in the time-domain exploit this form of Maxwell's equations. Taflove and Hagness [2] describe a conventional finite-difference time-domain (FDTD) scheme based on leap-frog time integration, which suffers from the staircase approximation inherent to the FDTD scheme formulated on structured (Cartesian) grids. Rodrigue and White [3] use hexahedral finite elements for the spatial discretization and time step the coupled Maxwell's equations with a leap-frog scheme, which gives a method that reduces to the FDTD scheme on rectilinear grids. The algorithm presented by Rodrigue and White [3] does not allow for unconditionally stable time-stepping and they do not consider tetrahedral meshes that are useful for local mesh refinement. Rieben, Rodrigue and White also published a similar method [4] that exploits higher order approximations for the spatial and temporal discretization. Riley and Jin [5] use finite-element (FE) techniques to discretize with respect to space and they arrive at a wave equation for the electric field that also involves the magnetic field. They use an explicit update algorithm of leap-frog type to compute the magnetic field by means of integration of Faraday's law. However, the time-stepping scheme used for the updating of the wave equation is not mentioned or described in their paper. Furthermore, their paper does not provide any information on the stability properties for their time-domain method.

A combination of the FDTD scheme applied to large homogeneous regions for efficiency and the finite-element method (FEM) for regions with complicated geometry and materials is attractive for many scattering problems. Wu and Itoh proposed FEM–FDTD hybridizations for both two [6] and three [7] dimensions. These schemes suffer from late-time instabilities that may be damped by temporal filtering [8]. Abenius et al. [9] combine the FDTD scheme with an implicit FEM and numerical studies indicate that it is stable, although no formal proof of stability is given. Monorchio et al. [10,11] proposed a hybrid that suffers from late-time instabilities and some attempts to mitigate this problem involve averaging or extrapolation techniques. Marrone and Mittra describe a way of interfacing triangles [12] and tetrahedrals [13] to FDTD cells but no explicit Courant criterion is derived. Rylander and Bondeson presented a stable FEM–FDTD hybrids [14,15] for 3D problems that are stable up to the Courant condition of the FDTD scheme, where convergence studies [15] and proofs of stability [15,16] are available in the literature. We emphasize that the treatment of magnetically lossy materials is not considered in any of these articles on hybrid FEM–FDTD algorithms.

There is a broad selection of numerical techniques that are formulated in the frequency domain [17,18], such as the method of moments (MoM) that may be accelerated by the multi-level fast multipole method (MLFMM) for electrically large scattering problems. The MoM is particularly efficient for scattering problems with many different incident angles. However, it provides the response for only one single frequency in contrast to time-domain field solvers that yield the response in a broad frequency-interval. The MoM can handle inhomogeneous materials and it may be formulated in the time domain, but such methods are computationally expensive and difficult to program. For problems that feature non-linear media, frequency domain methods in general and the MoM in particular are inappropriate, if not impossible, to use.

In this paper, we present a stable FEM–FDTD hybrid method for electromagnetic problems in two dimensions that feature complex geometry with materials that have both electric and magnetic losses. In contrast to what is available in the open literature on the FEM treatment of magnetic losses in combination with electric losses, our method is distinguished by a number of unique features: (i) an unconditionally stable time-stepping scheme based on FE techniques and Galerkin's method applied to the first order system of Ampère's and Faraday's law; (ii) a proof of stability for this type of implicit FEM; and (iii) a generalization of the stable FEM–FDTD hybrid [14] for 2D problems that is stable up to the Courant condition of the FDTD scheme. The unconditionally stable time-stepping scheme that we present in this paper, reduces to a special case of the conventional Newmark scheme [19] when it is applied to problems without magnetic losses. The FE techniques used to construct the unconditionally stable time-integration scheme offers the possibility to also treat dispersive materials [20]. In addition, we demonstrate that it is feasible to use the FEM–FDTD interface in regions that have both electric and magnetic losses. We would like to stress that the two-dimensional case is important for the design of wing profiles intended for stealth aircraft, and we will consider the corresponding 3D formulation in a future publication since the 3D Maxwell problem is significantly different from the 2D problem. For