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Test Method

Algorithmic methods of reference-line construction for estimating long-term strength of plastic pipe system

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A R T I C L E I N F O

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A B S T R A C T

Two simple and error free methods (direct and interpolation) for obtaining mathematical models for constructing reference lines were developed and successfully applied to the hydrostatic stress-rupture data of polyethylene pipes. Both methods employed an algorithmic process that analyzed the observed stress-rupture data along with its mathematical model of the 50% regression (LTHS) line. For each method, a shift value Δc was determined and was used to obtain the mathematical model for constructing reference lines that satisfied the requirement of ISO TS 26873. That is, the reference lines so constructed accommodated at least 97.5% of all stress-rupture data points on or above this line, in addition to being parallel to and vertically shifted below the 50% regression lines by an amount Δc. In the direct method, the reference line was made to pass directly over the data point that is equal to or the first data point greater than the 97.5% data position among all data points. On the other hand, the interpolation method extracted a shift value that corresponded to the 97.5% data position by interpolating between the first data points over and below 97.5%. In this case, the reference lines were made to pass through the interpolated position of 97.5% at every temperature. The advantage of the proposed algorithmic methods is that the determination of mathematical models for reference lines only involves finding the data position(s) with a vertical shift value of Δc that satisfies the ≤97.5% requirement. With these methods, uncertainties and errors associated with the current trial and error approach for constructing reference lines can be eliminated. In this paper, the details of the algorithmic process for obtaining a proper shift value and using it to develop the mathematical model are described for each method. Also, examples of constructing reference lines using these models are illustrated for polyethylene pipes.

1. Introduction

The use of polymeric materials in structural applications is increasing as these materials continue to demonstrate the appropriate properties necessary for their wider use. In particular, applications to pipe systems for conveying fluids under long-term internal pressures have been achieved with much success. In this regard, so-called "reference lines" [1] are made to provide ways to calculate the stress-rupture properties of various thermoplastic pipes [2,3]. In particular, mathematical models for reference lines have been useful in designing piping systems with an appropriate code of practice and standards. Thus, for applications where varying pressure and/or temperature are involved during the lifecycle of thermoplastic pipes, the reference lines are of much value in determining design pressures when used together with service coefficients and damage rules such as the Miner's rule [4–11]. An ISO Technical Specification [1] has been published that defines and describes the procedure for constructing reference lines. The reference line is defined as *"a mathematical description of the stress-rupture properties of thermoplastic materials, giving the expected hoop strength of the product with a certainty of at least 97.5%"*. Hence, the construction procedure is such that the 50% regression (predicted mean long-term hydrostatic strength – LTHS) line per ISO 9080 [12] is vertically adjusted downward until at least 97.5% of all stress-rupture data are on or above this line. In doing this, the current practice involves cross-adjusting two dependent parameters of the 50% regression (LTHS) equation by trial and error until the 97.5% requirement is met. This trial and error approach is cumbersome and can produce different results between those drafting reference lines. The proposed method eliminates these

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problems by using a straightforward algorithmic process [13] that determines the single shift factor for drawing reference lines. In light of the fact that some piping applications require butt fusion joints to be designed to accommodate varying pressure and temperature during their operations, the reference line concept can readily be adapted to provide relevant long-term test data for such design calculations. For this, an accurate and error free way of constructing reference lines, as described in this paper, would be of much value in treating such butt-fusion long term strength data.

2. Reference line construction

As per ISO TS 26873 [1], the reference lines are constructed by using the 50% regression equation of mean long-term hydrostatic strength (LTHS). This mathematical model is obtained by the method described in ISO 9080 and is given in Equation (1) [12].

$$\log(t) = c_{1i} + \frac{c_{2i}}{T} + c_{3i}\cdot\log(\sigma) + c_{4i}\frac{\log(\sigma)}{T} \tag{1}$$

Here, t is the expected time to failure in hours; σ is the hoop stress in megapascals; T is the temperature in Kelvins and c_{1i}, c_{2i}, c_{3i}, c_{4i} are the parameters of the model. *i* is 1 or 2 which represents two different failure mode branches (sometimes referred to as ductile and brittle failures) that may appear in the curve. These parameters are readily obtained by the least square error estimate method described in the annex of ISO 9080 [12].

Using Equation (1), the reference lines are produced by vertically shifting downward the LTHS lines by an amount Δc, until at least 97.5% of all failure points are on or above the shifted lines (Fig. 1). Thus, the reference line of each failure type takes the equation of the form given in Equation (2) [1].

$$\log(t) = A_i + \frac{B_i}{T} + c_i\cdot\log(\hat{\sigma}) + \frac{Di}{T}\cdot\log(\hat{\sigma}) \tag{2}$$

Here, A_i, B_i, C_i and D_i are parameters of the model and $\hat{\sigma}$ is the stress on reference lines.

The vertical shift from Equation (1) to Equation (2) is achieved by cross-adjusting parameters A_i and B_i, while C_i and D_i remain the same as c_{3i} and c_{4i}, respectively. That is, this vertical shift maintains the same slope between LTHS and reference lines. The cross-adjustment of parameters A_i and B_i is normally made by trial and error until the adjusted lines accommodate at least 97.5% all failure points on or above this line.

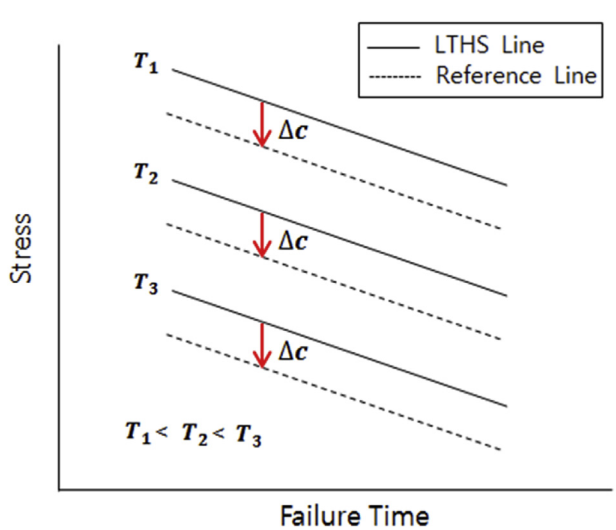

Fig. 1. Reference line construction from the stress-rupture LTHS regression line.

The trial and error way of manipulating LTHS equation to arrive at the proper reference line Equation (2) is cumbersome and can produce artificial differences among those performing the manipulation. The two algorithmic methods described below provide ways to find correct reference line equations in a straightforward manner without the shortcomings of the trial and error method.

2.1. Algorithmic method

To better understand the algorithmic method for determining the shift value, Δc, and therefore the reference lines (Fig. 1), Equation (1) is first considered.

In terms of stress as a dependent variable, Equation (1) can be written as Equation (3).

$$\log(\sigma) = \frac{\log(t) - \left(c_{1i} + \frac{c_{2i}}{T}\right)}{\left(c_{3i} + \frac{c_{4i}}{T}\right)} \tag{3}$$

To allow LTHS lines to shift downward vertically in the stress-rupture curve, at all temperatures in equal amount, a shift value Δc is introduced into Equation (3), and this is shown in Equation (4). Here, $\hat{\sigma}$ is the Δc shifted value of the stress.

$$\log\left(\hat{\sigma}\right) - \Delta c = \frac{\log(t) - \left(c_{1i} + \frac{c_{2i}}{T}\right)}{\left(c_{3i} + \frac{c_{4i}}{T}\right)} \tag{4}$$

Rewriting this in terms of failure times as the dependent variable, Equation (5) is obtained.

$$\begin{aligned}\log(t) &= \left(c_{1i}+\frac{c_{2i}}{T}\right) + \left(c_{3i}+\frac{c_{4i}}{T}\right)\cdot(\log(\hat{\sigma}) - \Delta c)\\ &= \left(c_{1i}+\frac{c_{2i}}{T}\right) - \left(c_{3i}+\frac{c_{4i}}{T}\right)\Delta c + \left(c_{3i}+\frac{c_{4i}}{T}\right)\cdot\log(\hat{\sigma})\\ &= (c_{1i} - c_{3i}\cdot\Delta c) + \frac{c_{2i} - c_{4i}\cdot\Delta c}{T} + c_{3i}\cdot\log(\hat{\sigma}) + \frac{c_{4i}}{T}\cdot\log(\hat{\sigma})\end{aligned} \tag{5}$$

Therefore, the construction of reference lines (per ISO TS 26873) now becomes identifying a line at each temperature, having shifted vertically by an amount Δc that accommodates at least 97.5% of the total stress-rupture data on or above these lines. Comparing Equations (5) and (2),

$$A_i = c_{1i} - c_{3i}\cdot\Delta c,\quad B_i = c_{2i} - c_{4i}\cdot\Delta c,\quad C_i = c_{3i}\quad \text{and}\quad D_i = c_{4i} \tag{6}$$

Hence, while the use of Equation (2) requires cross adjustment of dependent parameters A_i and B_i by a trial and error to arrive at the required reference lines, determination of Δc is only necessary when using Equation (5). To find the shift value Δc, following two algorithmic approaches are proposed.

2.1.1. Direct algorithmic approach

This approach involves first calculating the difference, e_i, between the LTHS stress (estimated stress), σ_i', and the observed stress, σ_i, values for all observed data points on the stress-rupture curve at all temperatures, as shown in Fig. 2. The difference, e_i, from all data set obtained is sorted in decreasing order and the e_i value of the data point that first corresponds to the 97.5% data position or above, of all data, is the value of Δc. For example, as illustrated in Fig. 2, when the e_i value of the *j*th data position (e_j) is the first data point less than the 97.5% data position and the $(j+1)^{th}$ data position is the first data point greater than 97.5% of all data, then the e_i value at $(j+1)^{th}$ position is the value of Δc. Therefore, in this approach, a reference line passes directly through the $(j+1)^{th}$