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Multifractal subgrid-scale modeling within a variational multiscale method for large-eddy simulation of turbulent flow

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ABSTRACT

Multifractal subgrid-scale modeling within a variational multiscale method is proposed for large-eddy simulation of turbulent flow. In the multifractal subgrid-scale modeling approach, the subgrid-scale velocity is evaluated from a multifractal description of the subgrid-scale vorticity, which is based on the multifractal scale similarity of gradient fields in turbulent flow. The multifractal subgrid-scale modeling approach is integrated into a variational multiscale formulation, which constitutes a new application of the variational multiscale concept. A focus of this study is on the application of the multifractal subgrid-scale modeling approach to wall-bounded turbulent flow. Therefore, a near-wall limit of the multifractal subgrid-scale modeling approach is derived in this work. The novel computational approach of multifractal subgrid-scale modeling within a variational multiscale formulation is applied to turbulent channel flow at various Reynolds numbers, turbulent flow over a backward-facing step and turbulent flow past a square-section cylinder, which are three of the most important and widely-used benchmark examples for wall-bounded turbulent flow. All results presented in this study confirm a very good performance of the proposed method. Compared to a dynamic Smagorinsky model and a residual-based variational multiscale method, improved results are obtained. Moreover, it is demonstrated that the subgrid-scale energy transfer incorporated by the proposed method very well approximates the expected energy transfer as obtained from appropriately filtered direct numerical simulation data. The computational cost is notably reduced compared to a dynamic Smagorinsky model and only marginally increased compared to a residual-based variational multiscale method.

1. Introduction

Turbulent flow exhibits a complex spatial distribution of eddies, which dictate the evolution of turbulence. An eddy is a structure, for instance, a tube or a sheet, that is formed by the local vorticity and its associated velocity field. The vortices in turbulent flows are then stretched and folded by their self-induced velocity fields as well as the velocity fields induced by all other vortical structures; see, e.g., [1] for elaboration. The repeated stretching and folding of the vorticity field as well as the strain rate represent a multiplicative process within the respective field. As shown in several studies, e.g., [2–4], gradient fields in turbulent flows such as kinetic energy dissipation and enstrophy exhibit multifractal scale similarity, enabling a novel approach to modeling turbulent flow.

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In Large-Eddy Simulation (LES) of turbulent flow, two modeling strategies are usually distinguished, as outlined, e.g., in [5]. On the one hand, functional models aim at modeling only the effect of the subgrid scales onto the resolved scales, but not necessarily their structure. The mostly-used functional approaches are subgrid-viscosity models. Structural models, on the other hand, aim to reconstruct or approximate subgrid scales directly. MultiFractal Subgrid-scale modeling (MFS) to estimate unresolved scales in LES was presented in comprehensive form in [6,7]. As turbulence is driven by vorticity and its self-induced velocity field, it was proposed by those authors to estimate the subgrid-scale velocity field from the subgrid-scale vorticity field via the law of Biot–Savart, where the integrated subgrid-scale vorticity field is reconstructed by a multiplicative process.

Multifractal structures originate from the repeated application of a scale-invariant multiplicative process on an initial field; see, e.g., [8,9]. Multiplicative processes are deterministic or stochastic multiplicative cascades. A scale-invariant distribution of multipliers maps the considered field in consecutive cascade steps from one cell to smaller subcells. The multiplier determines how the field of interest within one cell is distributed between the corresponding subcells. After a sufficient number of cascade steps, the resulting field becomes highly intermittent and displays multifractal scaling properties. The synthetic fields generated by the underlying multiplier distribution are statistically indistinguishable from the original field.

A first version of the multifractal subgrid-scale modeling approach was proposed in [10]. Based on Direct Numerical Simulation (DNS) of homogeneous isotropic turbulence, the authors showed that dissipation and enstrophy exhibit multifractal scale invariance over inertial-range scales. A further-developed version of the multifractal subgrid-scale modeling was presented in [6] and tested for homogeneous isotropic turbulence in the accompanying study [7]. Moreover, a backscatter limiter was introduced to control the stability of the numerical method. Those studies contain an exhaustive discussion of the energy transfer introduced by the multifractal subgrid-scale modeling approach based on extensive *a priori* and *a posteriori* testings, among others. The application of the multifractal subgrid-scale modeling to passive-scalar mixing problems was shown, e.g., in [11]. In that study, multifractal modeling of the subgrid scales was incorporated into a "non-linear Large-Eddy Simulation (nLES)" method, which constitutes an enhanced approach of multifractal subgrid-scale modeling with adaptive backscatter limiter. Therein, nLES is devoted to the evaluation of the subgrid-scale terms in their original non-linear form. Recently, the nLES was applied to further specific applications, e.g., in [12].

Further structural models approximating the subgrid-scale velocity are, for instance, the velocity-estimation model proposed in [13], where the unfiltered field is interpolated on a finer grid to generate some smaller scales, and the approximate deconvolution model presented in [14], which explicitly introduces a filter and its approximate inverse to obtain a representation of the subgrid-scale velocity. Scale-similarity type models, originally proposed in [15] and further generalized, e.g., in [16], are also relevant for the evolution of the present multifractal subgrid-scale modeling approach. In their original form, the subgrid-stress tensor is evaluated assuming equivalence between the smaller resolved and the subgrid scales. A model using fractal interpolation for the construction of a synthetic subgrid-scale velocity field was proposed in [17], and a model based on the subgrid-scale vorticity was presented, e.g., in [18]. In the latter study, the subgrid-scale structure of turbulence is assumed to consist of stretched vortices.

The variational multiscale approach to LES was originally introduced in [19] and later broadened in [20]; see also the reviews in [21,22]. In those studies, it was argued that the Variational Multiscale Method (VMM) provides an improved mathematical foundation for LES. Up to now, the VMM has either been used based on a three-scale separation including a subgrid-viscosity term acting only on the smaller resolved scales or as a residual-based two-scale version using the residual together with an appropriate parameter to approximate the unresolved-scale quantities. Recently, applications of the three-scale variational multiscale LES to turbulent incompressible flow and turbulent variable-density flow at low Mach number can be found in the particular form of an Algebraic Variational Multiscale-Multigrid Method (AVM^3), e.g., in [23,24], respectively. The residual-based variational multiscale LES was originally proposed for turbulent incompressible flow in [25,26], analyzed with respective to its dissipative properties in the context of turbulent incompressible flow, e.g., in [27,28], and later further developed for and applied to turbulent variable-density flow at low Mach number in [29]. An important aspect of the variational multiscale concept is that scales are separated by variational projection rather than by filtering. Particularly this feature offers a formulation that enables a straightforward introduction of structural subgrid-scale models into the "filtered/projected" Navier–Stokes equations.

In this study, we propose multifractal subgrid-scale modeling within a variational multiscale method, representing the first approach to integrate multifractal subgrid-scale modeling into the variational multiscale framework. This opens a new field of application of the variational multiscale framework in LES, as it is accounted for the introduction of structural turbulence models into the variational multiscale concept. Moreover, within a residual-based variational multiscale method, a backscatter limiter as proposed in [7] does not appear to be required. As in the AVM^3, scale separation to further decompose the resolved scales is performed via level-transfer operators from plain aggregation algebraic multigrid methods. In contrast to the AVM^3, however, multifractal subgrid-scale modeling is used instead of a rather simple subgrid-viscosity term based on the Smagorinsky model. Moreover, the multifractal subgrid-scale modeling approach is applied to wall-bounded turbulent flow for the first time. Therefore, an extension of the multifractal subgrid-scale modeling to account for near-wall effects is derived in this work. To the best of the author's knowledge, multifractal subgrid-scale modeling for wall-bounded turbulent flow and the development of an appropriate near-wall limit has not yet been shown in any publication. The high potential of the proposed multifractal subgrid-scale modeling within a variational multiscale method is demonstrated for three of the most important and widely-used benchmark examples for wall-bounded turbulent flow; that is, flow in a channel, over a backward-facing step and past a square-section cylinder.