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Test Method

New correction terms concerning three-point and four-point bending tests

Faustino Mujika*, Ainhoa Arrese, Itziar Adarraga, Usue Oses

Materials+Technologies Group, Department of Mechanical Engineering, Faculty of Engineering of Gipuzkoa, University of the Basque Country, Plaza de Europa 1, 20018 San Sebastián, Spain

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A B S T R A C T

The present study deals with the effect of the variation of the contact point between the support and load application rollers in three-point and four-point bending tests and the possible influence of horizontal reactions, under the assumption of small bending angles. Therefore, this study includes some factors that were not taken into account when the effect of horizontal displacements that generate span variation was investigated by the first author. The analytic approaches corresponding to three-point and four-point bending have been compared with experimental results that correspond to a unidirectional carbon/epoxy specimen. The flexural modulus obtained with the new approach in all experimental conditions is near constant. Moreover, the calculated load-displacement curves obtained with that modulus matches the experimental curves.

1. Introduction

When bending specimens are supported on and loaded by rollers, the contact points between the specimen and the rollers vary according to horizontal and vertical displacements. Due to horizontal displacements, the support and load span decreases in three-point and four-point bending tests. Furthermore, in a four-point bending test the load span increases. The effect of the vertical displacement is the translation of the beam as a whole. Timoshenko [1] included the effect of the variation of the support span in a three-point bending test, showing that the slope of the load-displacement curve increases as the load increases. Theocaris et al. [2] investigated the three-point bending test at large deflections including friction forces at the supports, axial forces along the beam and the effect of span shortening due to roller supports. Recently, Batista [3] provided a solution of the same problem in terms of Jacobi elliptical functions. The agreement with experimental results was very good even in the case of large defections and remarkable friction effects. The testing aspects studied in the case of three-point bending have not been analyzed analytically in the case of four point bending, but experimental comparisons between three-point and four-point bending results have been carried out [4–6]. Mujika [7], under the assumption of small bending angles, proposed an analytic approach related to the support and load span variations in order to explain the different results of the flexural modulus obtained by 3-point bending and 4-point bending tests. The effect of the vertical displacement of the contact point was assumed as a negligible term. Nevertheless, it has been shown recently that it is of the same order as the span reduction [8]. In order to assess the effect of the vertical displacement of the contact point, we carried out more experiments with different support and load application radii, presuming that the incorporation of the vertical displacement in the model would explain the behaviour of bending tests. Nevertheless, the results obtained showed that the flexural modulus varied abruptly for different roller radii. Moreover, that modulus varied in the same test depending on strain range considered In order to improve the model, we have included also the influence of the horizontal component of the reactions.

The aim of the present study is to analyze three-point bending and four-point bending tests under the assumption of small bending rotations, and to check them with experimental results obtained on a specimen in different testing conditions. According to ISO14125 [9] when the ratio maximum displacement-span is less than 0.1, displacements can be considered small. This is equivalent to a rotation angle of 0.3 rad (17°). In the experimental part of the present work maximum angles have been less than 0.2 rad (11°). Besides using different rollers, the modulus was determined in

* Corresponding author.
E-mail address: faustino.mujika@ehu.eus (F. Mujika).

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three strain ranges of the same load-displacement curve. The new aspects that have been included in the present study are:

1. An exact approach for the variation of the contact point between the specimen and the support rollers and between the specimen and the load application rollers. Afterwards, the first order approach is obtained.
2. The effect of the horizontal reaction due to the variation of the contact point, without taking into account friction effects.

2. Variation of the contact point

Fig. 1 shows a left support roller of radius R where C_0 is the contact point of the unloaded configuration and C_1 is the contact point of the loaded configuration during the test.

Assuming small angles, the horizontal and vertical displacements of the contact point are, respectively:

$$\delta_C^h = R\sin\theta = R\theta \qquad \delta_C^v = R(1-\cos\theta) = \frac{1}{2}R\theta^2 \tag{1}$$

In Eq. (1), the second order tem of θ in the vertical displacement has been retained, because the effects of δ_C^h and δ_C^v on the displacement of the specimen are of the same order. The term concerning vertical displacement has been included recently in three-point and five-point bending [8].

3. Three-point bending

3.1. Effects of bending, shear and the variation of the contact points

3.1.1. Exact solution

Fig. 2 shows a three-point bending test in the undeformed and deformed configurations.

The effect of local deformation will be included in the experimental displacement measured by the machine, after determining the stiffness of the testing system [8]. Then, the displacements analyzed in this section include bending and shear effects of the specimen and the vertical displacement of the contact point explained in the previous section:

$$\delta = \frac{PL^3}{4E_f wh^3} + \frac{3PL}{10Gwh} + \frac{1}{2}R\theta_{3p}^2 \tag{2}$$

where P is the applied load; E_f the flexural modulus; G the out-of plane shear modulus; w the width; h the thickness; L the span in the deformed configuration; R the radius of the support roller; and θ_{3p} the bending rotation of the specimen at supports. The maximum strain and the maximum bending angle are:

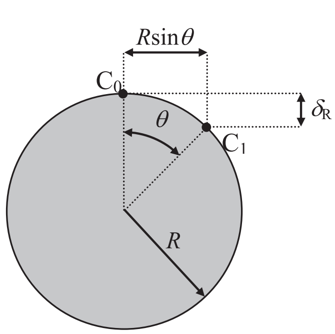

Fig. 1. Horizontal and vertical displacements of the contact point.

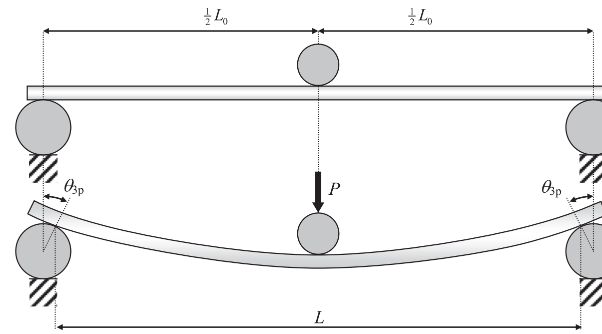

Fig. 2. Three-point bending in undeformed and deformed configurations.

$$\left.\begin{aligned}\theta_{3p} &= \frac{3PL^2}{4E_f wh^3}\\ \varepsilon_{3p} &= \frac{3PL}{2E_f wh^2}\end{aligned}\right\} \frac{\theta_{3p}}{\varepsilon_{3p}} = \frac{L}{2h} \tag{3}$$

According to Fig. 2 and taking into account Eq. (3), the span in the deformed configuration is:

$$L = L_0 - 2R\theta_{3p} = L_0 - Lf\varepsilon_{3p} \quad f = \frac{R}{h} \tag{4}$$

f being the radius-to-thickness ratio. From Eq. (4), the actual span is:

$$L = \frac{L_0}{(1+f\varepsilon_{3p})} \tag{5}$$

Resulting from Eqs. (3) and (5):

$$\varepsilon_{3p} = \varepsilon_{3p0}\frac{L}{L_0} = \frac{\varepsilon_{3p0}}{(1+f\varepsilon_{3p})} \tag{6}$$

ε_{3p0} being the strain that corresponds to the undeformed span, which is considered known. From Eq. (6), the actual value of the strain ε_{3p} is:

$$\varepsilon_{3p} = \frac{\varepsilon_{3p0}}{1+f\varepsilon_{3p}} \Rightarrow \varepsilon_{3p} = \frac{1}{2f}\left(\sqrt{1+4f\varepsilon_{3p0}} - 1\right) \tag{7}$$

After having determined ε_{3p} from Eq. (7), L can be calculated in Eq. (5) and θ_{3p} in Eq. (3). It is worth noting that, according to Eq. (7), given the strain of the undeformed configuration, the actual strain depends only on f.

3.1.2. First order approach

Considering a first order approach, the strain ε_{3pf} is obtained from Eq. (7), being:

$$\varepsilon_{3pf} = \varepsilon_{3p0}\left(1 - f\varepsilon_{3pf}\right) \Rightarrow \varepsilon_{3pf} = \frac{\varepsilon_{3p0}}{(1+f\varepsilon_{3p0})} \tag{8}$$

The span L_f is obtained considering the first order term in Eq. (5):

$$L_f = L_0\left(1 - f\varepsilon_{3pf}\right) \tag{9}$$

As a first order approach is considered, ε_{3pf} is used in Eq. (9). According to Eq. (9), Eq. (2) can be written as: