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A domain adaptive stochastic collocation approach for analysis of MEMS under uncertainties

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ABSTRACT

This work proposes a domain adaptive stochastic collocation approach for uncertainty quantification, suitable for effective handling of discontinuities or sharp variations in the random domain. The basic idea of the proposed methodology is to adaptively decompose the random domain into subdomains. Within each subdomain, a sparse grid interpolant is constructed using the classical Smolyak construction [S. Smolyak, Quadrature and interpolation formulas for tensor products of certain classes of functions, Soviet Math. Dokl. 4 (1963) 240–243], to approximate the stochastic solution locally. The adaptive strategy is governed by the hierarchical surpluses, which are computed as part of the interpolation procedure. These hierarchical surpluses then serve as an error indicator for each subdomain, and lead to subdivision whenever it becomes greater than a threshold value. The hierarchical surpluses also provide information about the more important dimensions, and accordingly the random elements can be split along those dimensions. The proposed adaptive approach is employed to quantify the effect of uncertainty in input parameters on the performance of micro-electromechanical systems (MEMS). Specifically, we study the effect of uncertain material properties and geometrical parameters on the pull-in behavior and actuation properties of a MEMS switch. Using the adaptive approach, we resolve the pull-in instability in MEMS switches. The results from the proposed approach are verified using Monte Carlo simulations and it is demonstrated that it computes the required statistics effectively.

1. Introduction

Micro-electromechanical systems (MEMS) have been used in widespread applications such as micro-switches, gyroscopes, accelerometers, etc. In order to design and analyze such devices it is required to accurately model the interaction of various physical fields such as mechanical, electrical and fluidic. In recent years, advances in numerical simulation methods have increased the ability to accurately model these devices [2–5]. These simulation methods assume that the material properties and various geometrical parameters of the device are known in a deterministic sense. However, low cost manufacturing processes used for MEMS often result in significant uncertainties in these parameters which may lead to large variation in the device performance. Thus, in order to design reliable and efficient electrostatic MEMS devices, it is required to quantify the effect of uncertain input parameters on various relevant performance parameters.

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Uncertainties can be described using stochastic quantities – uncertain parameters can be modeled using random variables and uncertain spatial functions are represented as random fields. Using this, the original governing equations can be reformulated as stochastic partial differential equations (SPDEs). Traditionally, sampling based methods such as Monte Carlo (MC) method has been used for systems with random input parameters. It involves generating various realizations of the input parameters according to the underlying probability distribution, and repeatedly employing the deterministic solver for each realization. Although the MC method is straightforward to implement and readily generates the required statistics, the simulations may become expensive as it offers slow convergence rate. Notably, the convergence rate for the MC method does not depend on the number of random dimensions or the smoothness of the stochastic solution in the random domain. The convergence of the MC method can be improved by using techniques such as the Latin hypercube sampling (LHS) [6], the quasi-Monte Carlo (QMC) method [7] and the Markov Chain Monte Carlo (MCMC) method [8], etc.

An important non-sampling approach is based on stochastic Galerkin method, where the basic idea is to project the unknown stochastic solution onto a stochastic space spanned by complete orthogonal polynomials. The stochastic Galerkin method was initially developed by Ghanem and Spanos [9] using Wiener–Hermite polynomial chaos expansion [10], where the orthogonal polynomials are chosen as global hermite polynomials in terms of Gaussian random variables. This idea was further generalized by Xiu and Karniadakis [11], to obtain exponentially converging algorithms even for non-Gaussian random variables. We developed a stochastic Lagrangian framework based on generalized polynomial chaos (GPC) in [12], to handle the uncertain electromechanical interaction. It was demonstrated that the stochastic framework can be effectively used to quantify the effect of uncertain input parameters on the performance of MEMS devices, as long as the solution is smooth in the random domain.

The stochastic Galerkin method provides high accuracy and faster convergence rate. However, as the number of stochastic dimensions of the problem increases, the number of basis functions needed to obtain accurate results increases rapidly, which reduces the efficiency. Also, the coupled nature of the deterministic equations that need to be solved to determine the modes of the solution makes the implementation non-trivial. It may be further complicated in situations when the governing equations take complicated form, such as nonlinear terms and coupled multiphysics.

In recent years, another class of methods known as stochastic collocation method [13–15] has been explored. The stochastic collocation method provides high resolution as stochastic Galerkin method, as well as easy implementation as the sampling based methods. This approach is based on approximating the unknown stochastic solution by constructing sparse grid interpolants in the multi-dimensional random domain, based on the Smolyak algorithm [1]. Using this algorithm, interpolation schemes can be constructed with orders of magnitude reduction in the number of support nodes to give the same level of approximation (up to a logarithmic factor) as the usual tensor product.

The stochastic Galerkin and collocation approaches provide fast converging approximations as compared to the sampling based methods, assuming that the unknown stochastic solution is sufficiently smooth in the random domain. However, in many physical systems, small variations in the uncertain parameters may lead to jumps in the solution. For example, in MEMS actuators, because of the nonlinear nature of the electrostatic actuation force, small variation in material properties and geometrical parameters may lead to a well known phenomenon known as *pull-in*. This pull-in instability is manifested as a discontinuity in the switch displacement in the random domain. In order to accurately compute the statistics of the stochastic solution in such situations, it is important to correctly capture these discontinuities in the random domain. To this end, several efforts have been made using the Galerkin approach, such as the wavelet based Weiner–Haar basis functions [16,17] and the multi-element GPC (ME-GPC) method [18,19]. The basic idea of ME-GPC is to adaptively decompose the random domain into a set of random elements, and then to employ a GPC expansion within each element to locally approximate the stochastic solution. This leads to a set of coupled deterministic equations that need to be solved within each random element. An adaptive sparse grid collocation methodology was presented in [15], based on the dimensional adaptive quadrature algorithm given in [20], to study the equilibrium jumps encountered during the stochastic modeling of natural convection problems. This approach automatically detects the more important dimensions and the sparse sampling is appropriately biased in those dimensions.

This work proposes a *domain adaptive stochastic collocation approach* to effectively handle discontinuities or sharp variations in the random domain. The basic idea of the proposed methodology is to adaptively decompose the random domain into subdomains. Within each subdomain, we then construct the sparse grid interpolant using the classical Smolyak construction in a hierarchical fashion, to approximate the stochastic solution locally. The adaptive strategy is governed by the *hierarchical surpluses*, which are computed as part of the interpolation procedure. These hierarchical surpluses then serve as an error indicator for each subdomain, and lead to subdivision whenever it becomes greater than a threshold value. The hierarchical surpluses also provide information about the more important dimensions, and accordingly the random elements can be split along those dimensions.

During the preparation of this manuscript, the authors came across two recent methods which also deal with problems with limited regularity in the stochastic domain. The first approach, multi-element probabilistic collocation method (ME-PCM) proposed by Foo et al. [21], discretizes the parametric space, and prescribes a collocation/cubature grid on each element. Although, both the ME-PCM method and our approach, adaptively decompose the parametric space into elements, the construction of the interpolant within each random element, and, more importantly, the computation of local error indicators, which ultimately leads to adaptive refinement, are significantly different. For the benefit of readers, we summarize the key differences between the two approaches as follows: