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Property modelling

Mullins effect modelling and experiment for anti-vibration systems

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ABSTRACT

An approach based on rebound energy (resilience) change is proposed to predict stabilisation of the Mullins effect for anti-vibration systems. A silicone rubber product manufactured in industry was selected for experimentation and verification. A Mullins indicator, in terms of the maximum loading forces over the accumulated residual deflection throughout the loading-unloading cycles, is proposed as a criterion to evaluate the stabilisation of the Mullins effect. Industry typically employs a three-loading/unloading-cycle routine on this silicone rubber product to remove the Mullins effect by approximately 75%. To achieve 95% accuracy for stabilisation, seven loading-unloading cycles are suggested. Verification shows that the proposed approach predicts results very close to measured experimental values, and the method can be used for engineering design and industrial applications.

1. Introduction

Polymer materials have damping capacity and energy absorption suitable for engineering applications. Rubber-to-metal-bonded systems are widely used for anti-vibration applications. In current design and industrial applications, the most important design parameters for rubber components, i.e., the stiffness and the fatigue requirements, are only referenced from the loading part of the loading-unloading history, Luo et al. [1–4]. Rubber-like materials exhibit an appreciable change in their mechanical properties during the loading-unloading process, especially in the first few cycles from a virgin state. This stress-softening phenomenon is referred to as the Mullins effect [5–7]. There are limited studies in the literature that consider this effect in the application and design of anti-vibration components used in industry. There is neither a well-defined approach to model the Mullins effect during the design process nor a suitable criterion to evaluate the effect on polymer components. Typical practice in industry is to load and unload polymer products three times to remove the Mullins effect before beginning product testing.

The concept of damage has been used by many scientists to model the Mullins effect. Ogden and Roxburgh [8,9] used a single additional (softening) variable to model the idealised Mullins effect. Dorfmann and Ogden [10] proposed two additional variables in the energy function to capture experimentally observed softening and residual strain response. Negative stress at zero deformation was a necessary pre-requisite to include residual strain in their

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model. Andrieux and Saanouni [11] proposed a three-dimensional strain model that accounted for the Mullins effect and irreversible strain. The model was limited to the prediction of incompressible hyperelastic behaviour with damage under tension on loading paths without unloading. Previously, a phenomenological continuum model for ethylene propylene diene monomer (EPDM) was formulated [12]. The model incorporated the loading-rate effect and the damage effect, which provided good predictions of the stress-strain behaviour of EPDM. Gornet et al. [13] developed a constitutive model and integrated it with a continuous damage approach to model the Mullins effect using Abaqus [14]. Other researchers investigated stress-softening behaviour using several styrene-butadiene rubbers (SBRs) with various amounts of fillers and different crosslink densities [15,16]. The authors observed that both the filler amount and the crosslink density affected cyclic softening of the SBRs. They proposed a model framework to account for both Mullins-induced residual strain and cyclic softening. Praffcke and Abraham [17] attempted to model actual applications of the Mullins effect by manually correcting the relevant parameters. They concluded that progressive stress-softening damage was not supported by the Abaqus Mullins effect model and that a more simplified approach would most likely provide a better cost-benefit ratio in many practical applications. Rickaby and Scott [18] determined that not all softening features would be relevant for a particular application and that some of them could be ignored to simplify simulations. Although the Mullins effect has been studied over many years, it is still recognised as a major obstacle in understanding the behaviour of rubber-like materials. Additionally, not all of the models mentioned above depend on parameters that can be compared to measurable physical quantities, as is the case for many other models [19,20].

Luo et al. [21–23] introduced rubber rebound energy (resilience) as a single function into a strain energy density expression. The rebound resilience, the ratio between the rebound energy and the initial loading energy, is a property of rubber materials and a parameter for a given rubber. They predicted loading-unloading responses as part of an idealised Mullins effect without residual strain.

In general, the modelling of the Mullins effect should include the residual strain and provide subsequent reloading-unloading capability as well as real parameters with engineering concepts. In this study, the work is extended to include residual strain and reloading-unloading predictions. In addition to the single function used in [21–23], two more functions are included in the strain energy model so that it can capture the observed softening and residual strain response simultaneously. Furthermore, the model can be used for calculating the reloading-unloading response that is different from the first loading and unloading cycle. Consequently, stabilisation of the Mullins effect is evaluated.

We first introduce the general form of constitutive models without the Mullins effect and the constitutive model for this approach. Then, we present a concept for rebound energy and its measurement, concluding with experimental validation following simulation of a rubber component.

2. Constitutive models for rubber material

2.1. Hyperelastic models of rubber material without stress softening

There are several hyperelastic material models that do not consider the Mullins effect. They are commonly used to describe rubber and other elastomeric materials based on strain energy potential or strain energy density, Ogden [24] and Bower [25]. These hyperelastic models for rubber material can be expressed in a general form:

$$W = W_I(\bar{I}) + W_J(J_{el}), \tag{1}$$

where $W_I(\bar{I})$ is the deviatoric part of the strain energy density of the primary material response and $W_J(J_{el})$ is the volumetric part of the strain energy density.

$\bar{I}$ can be further expanded into $\bar{I}_1$ and $\bar{I}_2$, which are the first and second deviatoric strain invariants defined as

$$\bar{I}_1 = \bar{\lambda}_1^2 + \bar{\lambda}_2^2 + \bar{\lambda}_3^2 \text{ and} \tag{2}$$

$$\bar{I}_2 = \bar{\lambda}_1^{-2} + \bar{\lambda}_2^{-2} + \bar{\lambda}_3^{-2}, \tag{3}$$

where $\bar{\lambda}_i$ are the deviatoric stretches.

J_{el} is the elastic volume ratio.

The corresponding stresses may be written as.

$$\sigma_{ij} = \frac{2}{J_{el}}\left[\frac{1}{J_{el}^{\frac{2}{3}}} + \left(\frac{\partial W}{\partial \bar{I}_1} + \bar{I}_1\frac{\partial W}{\partial \bar{I}_2}\right)B_{ij} - \left(\bar{I}_1\frac{\partial W}{\partial \bar{I}_2} + 2\bar{I}_2\frac{\partial W}{\partial \bar{I}_2}\right)\frac{\delta_{ij}}{3} - \frac{1}{J_{el}^{\frac{4}{3}}}\frac{\partial W}{\partial \bar{I}_2}B_{ik}B_{kj}\right] + \partial W \tag{4}$$

where B_{ij} is the component of the left Cauchy-Green deformation tensor $\boldsymbol{B}$, and δ_{ij} is the Kronecker delta.

More details on the above equations, including how these equations are obtained, can be found in [14,24,25]. Equation (1) has been widely used to predict material response under the loading portion of a loading-unloading process. However, it is not suitable for prediction of the response during the unloading portion or the reloading-unloading process. Equation (1) needs to be modified to account for unloading processes and the Mullins effect.

2.2. Rebound energy approach model with the Mullins effect

A function is added to modify the first part (W_I) of Equation (1) to account for the unloading and reloading conditions. Two more functions are added to Equation (1) to include residual strain. Hence, a constitutive model based on the rebound energy approach may be expressed in the form.

$$W = \theta_I(\beta)W_I(\bar{I}) + W_J(J_{el}) - \theta_r(\beta)R(\bar{I}_1, \xi, \beta) \tag{5}$$

where β is a nominal scale variable with a loading-unloading process. $\beta = 0$ when loading (including